TRIGONOMETRY

Made Easie;

Fitted to the

MEANEST CAPACITY;

IT BEING THE

FOUNDATION

OF

Astronomy, *Navigation*, *Dyalling*, *Surveying*, *Geography*, *Perspective* and *Gunnery*, &c.

By *John Holwell* Philomath:

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THE EPISTLE TO THE READER.

Courteous Reader,

I *Have here Preſented thee with a ſmall Piece of* Trigonometry; *which is no more than the making of it eaſie; ſo that any Perſon may come to the Knowledge thereof*

thereof with more eaſe than by former Authors. Much may be ſaid of this moſt ſublime Art; but many Authors have writ learnedly thereof; but when thoſe of a mean Capacity come to read them, they are as it were in a Wood, by reaſon of the Conſtructions of the Sines and Tangents, and that is always the firſt thing that they Treat of, and hath been the Cauſe that many that have had but a ſmall Stock of Euclid *in their Heads, could not come to the right knowledg thereof; therefore have I made this Eſſay, to make it eaſie without the Conſtruction of the Sines and Tangents, and Logarithmes; that thoſe that have but ſmall knowledge of the*

Ma-

Mathematicks, *may come to resolve all the Cases thereto belonging. So that the young Student may make a further Progress in other Arts, it being my aim to farther all such in these* Mathematical Studies; *as for the Learned, they have no need of being instructed by me: and kind Reader, if thou do find benefit by these my Labours, I hope thou wilt give it thy good word, and in so doing, thou wilt oblige thy Friend to*

Serve thee in what he may or can,

From my House on the East side of *Spittle-fields*, *Nov.* 6. 1684.

John Holwell.

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The PROEME.

THose that do intend to be Proficients in the Knowledge of Trigonometry, must not spare for pains in the attaining thereof; but the Ancients had far more trouble than there doth need now; for that Invention of my Lord Napier Baron of Merchiston, hath made it far more easie: For whereas the Ancients did multiply six or eight Figures together, now that is performed by Addition only; and instead of Division, now is used Substraction: and Mr. Briggs hath taken the pains from the Invention, to frame Tables of Logarithms to a great many thousands of absolute Numbers, and hath also made a Table of Artificial Sines and Tangents, which is no more than the Log. of the Natural; there being as yet found no proportion between a straight Line and the

Arch of a Circle; therefore the Ancients framed a Square without the Circle, and a Square within; which Squares do bear proportion one to the other; for the Diagonal being drawn through the Circles, cuts both the Squares in the Angles; by which means there will be four Iſoſcole Triangles, whoſe Baſes are the Tangent-Lines of the Circle, and the Square that is inſcribed in the Circle, is parallel to the Square that is circumſcribed, and is cut by the Diagonals, which Diagonals are called Secants, from their cutting of both the Squares; and by reaſon of two Squares being parallel one to the other, therefore are they proportional by the (2 *E.* 6) *for if there be drawn never ſo many Parallels to one ſide of any Triangle, then all the Segments of the ſides ſhall be proportional* (2 *E.* 6) *therefore the Square within the Circle, doth repreſent the Sines, and the Square that is without the Circle, doth repreſent the Tangents, and hath its Name from touching the Circle, and the Diagonals that do cut both the Squares, are called Secants, from their cutting through the Circle: And the Ancients have divided*

the

the Semi-diameter of the Circle in 10, 00000, *and* 90 *parallel Lines being drawn through the equal Divisions of the several degrees of the Circle to any Diameter, cut the other Diameter in unequal Parts; which Parts are placed in a Table, and are called Natural Sines; and a Line drawn from the Center of the Circle, through every degree of the Circle, until it cuts the Square without, the Divisions there also being unequal, are gathered into a Table, and are called Tangents; and the Lines that are drawn from the Center through the several Degrees of the Circle, till they cut the Tangent-Line without, make the Spaces of those Lines between the Circle and Tangent-Line also unequal, and are also gathered into a Table by the Ancients, and are called Secants; but those that please to make use of those Signs, Tangents and Secants, must multiply the second Term of the Proportion by the third, and that Product must be divided by the first, and the Quotient gives the fourth Term; and each Term doth consist of six Figures, if not more; and the last Term must be sought for in a Table also:*

But by that Noble Invention of my L. Napier, *we make uſe of no more Figures, and yet perform the ſame thing by Addition and Subſtraction; but the manner of making of both Natural Signs, Tangents and Secants, you may find in Mr.* Newton's Trigonometria Britanica, *and in many other Authors; it being my intent only to make an eaſie Explanation of the ſeveral Caſes, with the Reaſon of the framing of their Proportions, and ſhall refer you for the Tables of Sines & Tangents, to my* Sure Guide to the Practical Surveyor; *where you have the Sines and Tangents to every five Minutes; but if you require more Exactneſs, you may uſe* Norwood's Epitome, *or* Gellibrand's Epitome, *both Books being but of ſmall price.*

TRI-

TRIGONOMETRY
Explained.

CHAP. I.

Of some Things necessary to be understood in the Practice of Trigonometry.

1. ALL plain Triangles consist of six parts (*viz.*) three sides, and three Angles.

2. The Angles of all plain Triangles are measured by a Line of Chords described on the angular Points; as the Arch D E, in *Fig.* 1. is the measure of the Angle A.

3. the Sides of a plain Triangle are measured with a Line of equal parts.

4. All Circles are divided into 360 deg. and each degree into 60 parts, called Minutes.

5. The three Angles of any plain Triangle is equal to 180 deg. (by the 32 of the first Book of *Euclids* Elements.)

6. A Triangle is either right angled, or obtique angled; that is right angled, that hath one Angle in it equal to 90 deg. according to the 26th. Definition of the first of *Euclid*, as in *Fig.* 1. the Triangle A B C is right angled at B, and the other two Angles A and C are also equal to 90 deg. because the three Angles are equal to 180 deg. (32 E. 1.) the side that doth subtend to the right, is called the Hypothenusal, and the Side A B is call'd the Base, and B C the Perpendicular.

7. An oblique Triangle is that which hath one Angle greater than 90 deg. and the side that doth subtend the greater Angle here, is called the Base, and the obtuse Angle is in most Authors the Angle C, the Angle A and B being both acute; as in *Fig.* 2.

8. The Ancients have made 12 Cases; for the solution of which, they have framed four Axioms, or general Rules.

AXIOM I.

In all plain right angled Triangles, any side may be made the Radius, and the other side will be Sines, Tangents or Secants; and look what

what proportion the ſide put for Radius, hath to the true Radius or Sine of 90 deg. the ſame proportion have the other ſides to the Sines, Tangents or Secants by them repreſented.

The Explanation of this Axiom,

The making of either ſide the Radius, is to make it the Semi-diameter of a Circle, and then the other ſides become Tang. or Secants, or elſe all Sines (*viz.*) if the Hypothenuſal be made the Semi-diameter, then the Baſe and Perpendicular falls within the Circle, and therefore become Sines, (*viz.*) the Baſe becomes the made ſine of the Angle at the Perpendicular, and the Perpendicular becomes the made ſine of the Angle at the Baſe, as may be ſeen *Fig.* 3. But if the Baſe be made the Radius, the Perpendicular becomes the made Tangent of the Angle at the Baſe, and the Hypothenuſal becomes the Secant of the ſame Angle; and if the Perpendicular be made the Radius (or Semi-diameter of a Circle) then the Baſe becomes the Tangent of the Angle of the Perpendicnlar, and the Hypothenuſal becomes the Secant of the ſame Angle; as may be ſeen in *Fig.* 4, & 5. The fourth is, where the Baſe is made the Radius; and the fifth is, when the Perpendicular is

made the Radius; and this serves for all the right angled Cases, which are in number Six.

AXIOM II

In all plain Triangles the sides are in such proportion one to the other, as are the sines of their opposite Angles.

AXIOM III.

In all plain Triangles the Sum of the sides containing the Angle given, bears the same proportion to the difference of the said two sides, as doth the Tangent of half the Sum of the two opposite Angles to the Tangent of the difference of the same two Angles above or under the half Sum.

The Explanation of this Axiom.

Construction.

Let A B C in *Fig.* 6. be an oblique Triangle; make A E and A D equal to A C, and draw C E and parallel thereto draw B G, and draw a Line from D to C, extending, if need be.

De-

Demonstration.

Foraſmuch as A E is equal to A C, therefore (by the 5th. of the 1ſt. Book of *Euclid*,) the Angle A E C is equal to A C E, and by the ſame reaſon the Angle A D C is equal to the Angle A C D; therefore the whole Angle E C D is equal to both theſe Angles D E C, and E D C; therefore by the Corol. of the 31 Prop. of the 3d. Book of *Euclid*, the Angle E C D is a right Angle; and foraſmuch as B G is parallel to E C, therefore by the (29 *E.* 1.) the Angle B G C is alſo a right Angle, for it is equal to E C D, and the Angle D B G is equal to D E C, and the Angle B C E is equal to C B G, and by the (32 *E.* 1) the outward Angle A E C is equal to the two inward Angles E B C and E C B, put A C E common to both, then theſe two A E C and E C A are equal to theſe two Angles A B C and A C B; therefore either of theſe two Angles A E C and A C E is half the ſum of theſe two Angles A B C and A C B; therefore alſo the Angle E B G is half the ſum of the ſame Angles A B C and A C B.

Now if on one of the ſides of a Triangle there be drawn a parallel, it divides the other ſide proportional by the (18 *E.* 6) therefore D E is in proportion to E B, as

D C

D C is to C G; therefore also compoſed by (18 E. 5) as D B is to E B, ſo is D G to C G, that is,

As D B the Sum of the Sides A B and A C,

Is to their Difference E B,

So is D G the Tangent of half the Angles unknown,

To C G, the Tangent of the Angle C B G. it being that which the Angle A C B comes ſhort of the half Sum, as E C B is the exceſs of the half ſum above the Angle A B C; therefore as the ſum of the ſides is to their difference, ſo is the Tangent of half the ſum of the Angles unknown, to the Tangent of their difference above or under the half ſum, which was to be demonſtrated.

AXIOM IV.

In all plain Triangles, as the Baſe is in proportion to the ſum of the other two ſides, ſo is the difference of the two ſaid ſides, to the difference of the Segments of the Baſe.

The

The Explanation of this Axiom.

Conſtruction.

Let B C D be a Triangle given (as *Fig.* 7.) let C D be the Baſe, B D the ſhorteſt ſide upon the point B; with the diſtance of B D deſcribe the Circle A D F H, and let the ſide B C be produced to A; then is C A the ſum of the two ſides, becauſe B A and B D are equal; C H is the difference of the two ſides, and C F is the difference of the Segments of the Baſe.

Demonſtration.

If you draw the Lines A F and H D, the Triangles C H D and C F A, ſhall be equiangled, becauſe of their common Angle A C E, or H C D, and their equal Angles C A F and H D C, which are equal, becauſe the greater Arch H F, is the double meaſure to them both; and therefore as C D is to C A, ſo is C H to C F; which was to be demonſtrated: Hence it is evident that the three ſides of any plane oblique Triangle being given, the reaſon of the Angles is alſo given.

If

If any Angle exceed 90 Degrees, ſubſtract it out of 180 Degrees, and work with the Sine of the Remainer.

CHAP. II.

Containing the Menſuration of right Angled plain Triangles.

Caſe 1.

The Hypothenuſal and Angles being given to find the Baſe:

LET the Hypothenuſal be 500 equal parts, and let the Angle at the Baſe, be 30 deg. the Angle at C will be 60 deg. By the 6th. Conſideration of *Chap.* 1.

For the Laying down this Triangle.

Firſt, Draw your Baſe at pleaſure; then take off 60 deg. from off your Line of Chords, and placing one Foot of your Compaſſes in the point A, with the other deſcribe the prickt Arch DE; then take the quantity of your given Angle A, from the ſame line of Chords,

Chords, and place it from D to B; then draw A C, and make it equal to 100 equal parts; then from the point C, let fall a perpendicular to the Base A B; and then the Triangle is laid down.

Fig. 1.

First, *For the Finding the Length of the Base making the Hypothenuſal the Radius, the Proportion is thus;*

As the true Radius or Sine of 90 deg. B,	10, 000000
Is to the Log. of the made Radius the Hypothenuſal, A C—	2, 698970
So is the true Sine of the Angle at the Perpendicular, B C—	9, 837531
To the Log. of the made Sine the Base, A B—	12636501

The General Rule for working Proportion by Logarithms.

You muſt add the *Log.* of the ſecond and third Numbers together, and from their Sum ſubſtract the log. of the firſt, and the Remainer is the log. of the Fourth ſought, as doth appear by theſe following Operations. The Work may be abbreviated when the Radius

Radius is not in the First place, by taking the Complement Arithmetical of the log. of the First Number, and then adding the log. of the Second and Third, and the Complement Arithmetical of the First into one Sum; from which abating the Radius (or an Unite towards the left hand) the remainer is the log. of the Fourth Number.

The readiest way to find the Complement Arithmetical of any, is that of Mr. *Norwood*'s, in his *Doctrine of Triangles*; which is thus; to take the residue of the First Figure towards the left hand, unto 9; and so of the rest, until you come to the last Figure towards the right hand thereof; and for the last Figure, you must set down the residue to 10:

Example.

I would know the Complement Arithmetical of the sine of 40 deg. which is 9,808067, the residue of the first Figure to the left hand to 9, is 0, of 8, log. is 1, 0 to 9, is 9, of 8, to 9, is 1, of 0, to 9, is 9, of 6, to 9, is 3: So the Complement Arithmetical is 0191933. Mark this Rule well; for you will find it of good Use, and it is easily done, as you see.

But if the Radius be in the first place, then there is no need for the taking of the Comp. Arith. of the first Number; only you must

cut

cut off the first 1 to the left hand thus, x and you will have your Desire; as you may see in the foregoing Operation.

Fig. 4.

For the bare making the Base the Radius.

As the true Secant of the Angle at the Base, 30 *d.* A. *Co. Ar.* —	9937531
Is to the made Secant the Hypothenusal 500 A C. —	2698970
So is the true Radius or Sine of of 90 deg. B. —	10000000
To the Log. of the same Radius, the Base A B —	12636501

Fig. 5.

For the Base making the Perpendicular the Radius.

As the true Secant of the Angle at the Perp. C. 60 *d. Co. Ar.* —	19698970
Is to the log. of the made Secant the Hypoth. A C 500 —	2698970
So is the true Tangent of the Ang. at the Perpendic. C 60 *d.* —	10238561
To the log. of the made Tangent (*viz.*) the Base A B 433 —	2636501

Note,

Note, If the Hypothenuſal be made the radius, and a ſide be demanded, an Angle muſt go in the firſt Term of the Proportion, but it muſt be that Angle that is oppoſite to a ſide given; the ſide given muſt be in the ſecond Term, the Angle oppoſite to the ſide demanded, muſt be the Fourth Term.

But if an Angle be demanded, a ſide muſt be the firſt Term in the Proportion, and it muſt be that ſide that is oppoſite to an Angle given; the Angle given muſt be in the ſecond Term; the ſide oppoſite to the Angle demanded, muſt be the third Term, and the Angle demanded muſt be the Fourth Term.

But if the Baſe be made the Radius, and the Angle at the Baſe demanded, then muſt there be either the Hypothenuſal, or Perpendicular given beſides the Baſe: then muſt the made radius be firſt Term in the Proportion, the true radius muſt be the ſecond Term, the made Tangent, if the Perpendicular be given, or the made Secant, if the Hypothenuſal be given, muſt be in the third Term; and the Angle demanded muſt be the fourth Term: it muſt come out in the ſame denomination as the third Term (*viz.*) a Tangent or Secant.

But if the made radius be demanded, then the true Tangent or Secant of the Angle given, muſt be the firſt Term in the Proportion,

tion, the made Tangent or Secant must be the ſecond Term, the true Radius muſt be the third Term, and the made Radius muſt be the fourth Term.

But if the Perpendicular be made the Radius, and the Angle at the Perpendicular be demanded, then muſt either the Hypothenuſal or Baſe be alſo given; then the made Radius muſt be the firſt Term in the proportion, the true Radius muſt be the ſecond Term, the made Tangent or Secant muſt be the third Term, and the true Tangent or Secant muſt be the fourth Term.

But if the made Radius be demanded, the Angle at the Perpendicular with either the Baſe or Hypothenuſal will be given, then the true Tangent or Secant muſt be the firſt Term in the proportion, the made Tang. or Secant muſt be the ſecond Term, the true Rad. muſt be the third Term, and the made Radius will be the fourth Term.

Theſe Rules are the true Foundation of the varying the Proportions of all the Caſes of right angled plain Triangles; which ſhall be cleared by Examples in the following Caſes.

Case 2.

The Hypothenusal and Angles being given, to find the Perpendicular.

Let the Hypothenusal be 500 equal parts, and let the Angle at the Base, be 30 *d.* the Angle at C, will be 60 *d.* by the 1xth Consideration of the 1st. Chap. The Laying down of this Triangle is the same with the former.

Fig. 1.

For the Perpendicular making the Hypothenusal the Radius.

As the true Radius or Sine of 90 deg. B, ——— 10, 000000
Is to the Log. of the made Radius the Hypothenusal, A C 500 ——— 269897
So is the true Sine of the Angle at the Base, A 30 *d.* ——— 969897

To the Log. of the made Sine the Perpendicular BC 250 ——— 1239794

Fig

Fig. 2.

For the Perpendicular making the Base Radius.

As the true Secant of the Angle at the Base, A, 30 *d. Co. Ar.*—	9937531
Is to the Log. of the made Secant the Hypothenusal 500 A C.	2698970
So is the true Tangent of the Angle at the Base, A 30,——	9761439
To the Log. of the made Tangent the Perpend. B C 250,—	2397940

Fig. 3.

For the Perpendicular making the Perpendicular Radius.

As the true Secant of the Angle at the Perp. C. 60 *d. Co. Ar.*—	9698970
Is to the log. of the made Secant the Hypoth. A C 500——	2698970
So is the true Rad. or Sine of 90 *d.*	10000000
To the log. of the made Radius, the Perpendicular B C, 250,	12397940

Case 3.

The Base and Angles being given to find the Hypothenusal and Perpendicular,

Let the Base be 400, and the Angle at the Base be 30 deg. the Angle at C, being 60 *d.* according to the sixth Consideration of the 1st. Chapter.

For the Laying down this Triangle.

First, Draw your Base at pleasure; then set off the given distance from A to B, then on the Point B raise a Perpendicular; and take off 60 deg. from off your Line of Chords and placing one Foot of your Compasses in the point A, with the other describe the prickt Arch D E; then take the quantity of your given Angle from the same line of Chords, and place it from D to E; then draw a Line from A, through E, until it cuts B C, in C, and then you have done.

Fig.

Fig. 1.

For the Perpendicular making the Hypothenusa the Radius.

As the true Sine of the Angle at the Perpendicular C, 60 *d.* *C. A.*	0062469
Is to the Log. of the made Sine the Baſe A B, 400	2601059
So is the true Sine of the Angle at the Baſe A, 30 *d.*	9698970
To the Log. of the made Sine, the Perpendicular C B, 231,	2363498

Fig. 2.

For the Perpendicular making the Baſe the Radius.

As the true Radius or Sine of 90 deg.	10, 000000
Is to the Log. of the made Radius the Baſe, A B, 400,	2602059
So is the true Tangent of the Angle at Baſe A, 30 *d.*	9761439
To the Log. of the made Tangent the Perpend. B C, 231,	2363498

Fig. 3.

For the Perpendicular making the Perpendicular the Radius.

As the true Tangent of the Angle at the Perpend. C 60 *d.* C. A.	9761439
So is the Log. of the made Tangent the Base A B 400, ——	2602059
So is the true Radius or Sine of 90 *d.* B, ——	10000000
To the Log. of the made Radius the Perpendicular B C, 231—	2363498

Fig. 1.

For the Hypothenusal making the Hypothenusal the Radius.

As the true Sine of the Angle at the Perpendicular C, 60 *d.* C. A.	0062469
Is to the Log. of the made Sine the Base, A B, 400-- ——	2602059
So is the true Radius or Sine of 90 *d.* B,-- ——	10000000
To the Log. of the made Radius the Hypothenusal A C, 260— ——	12664528

Fig.

Fig. 2.

For the Hypothenuſal making the Baſe the Radius.

As the true Radius or Sine of 90 *d.* B.	10, 0000000
Is to the Log. of the made Radius the Baſe, A B, 400	2602059
So is the true Secant of the Angle at the Baſe A, 30 *d.*	0062469
To the Log. of the made Secant the Hypothenuſal A C, 462	2664528

Fig. 3.

For the Hyyothenuſal making the Perpendicular the Radius.

As the true Tangent of the Angle at the Perpendicular C, 60 *d. C. A.*	9761439
Is to the Log. of the made Tang. the Baſe A B 400	2602059
So is the true Secant of the Angle at the Perpend. C, 60 *d.*	0301030
To the Log. of the made Secant, the Hypothenuſal A C, 462	2664528

Caſe

Case 4.

The Perpendicular and Angles being given, to find the Base and Hypothenusal.

Let the Perpendicular be 300 equal Parts, and the Angle at the Base 30 *d.* and then the Angle at the Perpendicular will be 60 *d.* according to the 6th. Consideration in Chap. 1.

For the Laying down this Triangle.

First, Draw your Base at pleasure, and at the Point B, raise a Perpendicular; then make B C equal to 300 equal parts; then take 60 *d.* from your Line of Chords, and setting one foot of your Compasses in the Point C, with the other describe the Arch D E; then make D E equal to the Chord of 60 *d.* the Angle at the Perpendicular, and draw a Line from C, through E, until it intersect the Base A B, in A, and it is done.

Fig. 1.

For the Base making the Hypothenusal the Radius.

As the true Sine of the Angle at the Base A 30 *d. C. A.* ——	0301030
Is to the Log. of the made Sine the Perpendic. B C, 300, ——	2477121

So

So is the true Sine of the Angle at the Perpendicular C, 60 *d.* — 9937531

To the made Sine, the Base A B, 519 equal parts. — 2715682

Fig. 2.

For the Base making the Base the Radius.

As the true Tangent of the Angle at the Base A, 30 *d. C. A.*— 0238561

Is to the Log. of the made Tang. the Perpendicular B C, 300,— 2477121

So is the true Radius or Sine of 90 deg. B, — 10000000

To the made Radius, the Base A B, 519 equal parts. — 2715682

Fig. 3.

For the Base making the Perpendicular the Radius.

As the true Radius or Sine of 90 *d.* B, — 10,0000000

Is to the Log. of the made Rad. the Perpend. B C, 300 — 2477121

So is the true Tangent of the Angle at the Perpend. C, 60 *d.*— 0238561

To the Log. of the made Tang. the Base A B, 919. — 2715682

Fig.

Fig. 1.

For the Hypothenusal making the Hypothenusa the Radius.

As the true Sine of the Angle at the Base A, 30 *d. C. A.*	0301030
Is to the Log. of the made Sine the Perpendicular 300, B C	2477121
So is the true Radius or Sine of 90 *d.* B,	10000000
To the Log. of the made Radius A C, the Hypothenusal, which is 600 equal parts,	2778151

Fig. 2.

For the Hypothenusal making the Base the Radius.

As the true Tangent of the Angle at the Base A, 30 *d. C. A.*	0238561
Is to the Log. of the made Tangent the Perpend. B C, 300 equal parts	2477121
So is the true Secant of the Angle at the Base 30 *d.*	0062406
To the Log. of the made Secant the Hypothenusal A C, 600	2778151

Fig

Fig. 3.

For the Hypothenuſal making the Perpendicular the Radius.

As the true Radius or Sine of 90 *d.*	10,000000
Is to the Log. of the made Radius the Perpendicular 300,—	2477121
So is the true Secant of the Angle at the Perpendicular 60 *d.*——	0301030
To the Log. of the made Secant, the Hypothenuſal 600.——	2778151

Caſe 5.

The Hypothenuſal and Baſe being given, to find the Angles.

Let the Hypothenuſal be 500 equal parts, and the Baſe 400.

For the laying down of this Triangle.

Firſt, Draw your Baſe A B at pleaſure, and ſet the length thereof from A to B; then at the Point B, raiſe a Perpendicular; then take the length of your Hypothenuſal from the ſcale of equal parts, and ſet one point of your Compaſſes in the point A; with the other

croſe the Perpendicular at the point C, then draw the Line A C, and you have done.

Fig. 1.

For the Angle at the Baſe making the Hypothenuſal the Radius.

As the Log. of the made Radius the Hypothenuſ. AC 500 C A,	730103
Is to the true Radius, or Sine of 90 *d.*	10, 000000
So is the Log. of the made Sine the Baſe A B 400,	2602059
To the true Sine of the Angle at the Perpendiclar, 56 *d.* 16 *m.*	9903089

Which Angle being ſubſtracted from 90 deg. leaves the Angle at the Baſe A.

Fig. 2.

For the Angle at the Baſe, making the Baſe the Radius.

As the Log. of the made Radius, the Baſe 400	7397941
Is to the true Radius, or the Sine of 90 *d.*	10, 000000
So is the Log. of the made Secant the Hypothenuſal	2698970
To the true Secant, the Angle at the Baſe A, 33 44	10096911

Caſe

Case 6.

The Hypothenuſal and Perpendicular being given, to find the Angle.

Let the Hypothenuſal be 500 equal parts, and the Perpendicular be 300.

For the laying down of this Triangle,

Firſt, Draw your Baſe at pleaſure, and at the Point B, raiſe your Perpendicular, which make equal to 300 equal parts; then take 500 equal parts from your Scale, and ſetting one foot of your Compaſs in the Point C, and where the other foot of your Compaſſes toucheth the Point, the Baſe will be A; then draw the Hypothenuſal A C, and the Angle is laid down.

Fig. 1.

For the Angle at the Baſe, making the Hypothenuſal the Radius.

As the Log. of the made Radius the Hypothenuſal 500 A C,— 7301030
Is to the true Radius or Sine of 90 *d.* —————— 10,000000

So

So is the Log. of the made Sine the Perpendicular B C 300, — 2477121

To the true Sine, the Angle at Base A, 36 *d.* 52 *m.* — 9778151

Fig. 3.

For the Angle at the Perpendicular, making the Perpendicular the Radius.

As the Log. of the made Radius the Perpendicular B C, 300 — 17522879
Is to the true Radius or Sine of 90 *d.* — 10000000
So is the Log. of the made Secant the Hypothenusal A C, 500 — 2698970
To the true Secant of the Angle at the Perpendicular C, 53. 8. 10221849

Case 7.

The Base and Perpendicular being given, to find an Angle.

Let the Base be 400 equal Parts, and the Perpendicular 300.

For the laying down of this Triangle.

First, Draw your Base at pleasure, and make

A B

A B equal to 400 equal parts, and at the Point B raiſe a Perpendicular, and make B C equal to 300 parts; then draw your Hypothenuſal A C, and the Triangle is laid down.

Fig. 2.

For the Angle at the Baſe, making the Baſe the Radius.

As the Log. of the Baſe, or the made Radius A B 400,
Is to the true Radius or Sine of 90 *d.*
So is the Log. of the made Tangent the Perpendicular B C,
To the true Tangent of the Angle at the Baſe A.

Fig. 3.

For the Angle at the Perpendicular, making the Perpendicular the Radius.

As the Log. of the Perpendicular
Is to the true Radius,
So is the Log. of the made Tangent the Baſe
To the true Tangent of the Angle at the Perpendicular.

For

Fig. 1.

For the Hypothenusal making the Hypothenusal the Radius.

As the true Sine of the Angle at the Base,
Is to the Log. of the made Sine the Perpendicular;
So is the Radius or Sine of 90 *d.*
To the Log. of the made Radius, the Hypothenusal.

Fig. 2.

For the Hypothenusal making the Base the Radius.

As the true Radius or Sine of 90 *d.*
Is to the Log. of the made Radius the Base,
So is the true Secant of the Angle at the Base
To the Log. of the made Secant the Hypothenusal.

Case 8.

Of Oblique Plain Triangles.

Fig. 4.

Two Sides of an oblique Triangle, with an Angle oppos'd to either of them, being given, to find the rest of the Parts in the Triangle

angle A B C, the ſide A B, be 100 equal parts, and the ſide A C be 70, and let the Angle at B, be 30 deg.

For the Laying down of this Triangle.

Firſt, Draw the Baſe A B, and make it equal to 100 equal parts; then at the end of the Baſe at B, lay down the quantity of your given Angle 30 *d.* and then draw a line as if it were to be an Hypothenuſal; then take off 70 equal parts from your Scale, and ſet one foot of your Compaſſes in the baſe line at A, and mark where the other doth touch the line B C; and from that place to the end of the Baſe A, draw a line, and you have done. For the finding of theſe Parts that are to be found, you muſt have two Operations; the firſt muſt be for the Angle at C; and the other for the length of the ſide B C.

And firſt, for the Angle C.

As the log. of the ſide A C, 70 equal parts Co. Ar.
Is to the ſine of the Angle B. 30 *d.*
So is the log. of the ſide A B, 100 equal parts
To the ſine of an Angle, 45 *d.* 35 *m.*

Which Angle being ſubſtracted out of

180 *d.* leaves the Angle (*viz.*) C, (*viz.*) 45 *d.* 35 *m.* from 180 *d.* leaves 134 *d.* 25 *m.* for the the Angle C.

For the side B C, first, you must add the Angle at C, to the Angle at B; and their sum substracted from 180 *d.* leaves the Angle at A.

Then say,

As the sine of the Angle at B,
Is to the log. of the side A C,
So is the sine of the Angle at A,
To the log. of the side B C.

Case 9.

Fig. 4.

Two Angles of an oblique Triangle being given, and a side opposite to either of them, to find the rest of the Parts thereof:

Let the Angle at A, be 15 *d.* 35 *m.* and the Angle at B, 30 *d.* and the side A B to be 100 equal Parts.

For the Laying down this Triangle.

First, you must draw the Base A B, and set off 100 equal Parts from A to B; then upon the Point A, lay down the quantity of the Angle at A, equal to 15 *d.* 35 *m.* and then

then on the on the Point B, lay down the quantity of the Angle at B, equal to 30 deg. and where the two lines do interſect, will be the Angle at C; and then you have done.

For the finding of the reſt of the parts of this Triangle, it is in the ſame manner as the laſt, only inſtead of ſides being in the firſt place, here muſt go angles; and then the proportion will run thus;

As the Sine of the angle C,
Is to the log. of the ſide A B,
So is the ſine of the angle at A,
To log. of the ſide C B.

For the Side A C, *it runs thus*;

As the ſine of the angle at A,
Is to the log. of the ſide C B,
So is the ſine of the angle at B,
To the log. of the ſide A C.

Caſe 10.

Fig. 4.

Two ſides of an Oblique Triangle which the Angle contained between them, being given, to find the other angles and ſide; let the ſide A B, be two equal parts, and the

ſide A C, 80 of the ſame parts, and the contained angle at A, be 40 deg.

For the laying down of this Triangle,

Firſt, Draw a Baſe-line, and make A B equal to 100 equal Parts, the length of the given baſe; then lay down the quantity of the angle at A, upon the point A; then draw the line A C, and make it equal to 80 *d.* of the ſame parts, as you did the line A B: then from the line C, and B, draw the line C B, and you have done. When you have laid it down, you will find that you will have the ſide C B, to find the angle B and C. And firſt, for either of the angles, you muſt do thus; Subſtract the given angle from 180 *d.* and the remainer will be the ſum of the two angles unknown; and you muſt take half of that ſum, and then add both of the two ſides together into one ſum, and ſubſtract one ſide out of the other, and note their Difference.

Then ſay,

As the log. of the ſum of the two ſides A B, and A C,

Is to the log. of their difference,

So is the tangent of half the ſum of the angles unknown, B and C.

To the tangent of the difference of the ſaid two angles above or under the half ſum.

There-

Therefore when you have found the rence of these two Angles above or under the half sum, if you add this difference to the half sum, that will be the angle at C; and if you substract it, there will remain the angle at B.

Then you may find the Side BC *thus*;

As the sine of the angle B,
Is to the log. of the side AC,
So is the sine of the angle at A,
To the log. of the side BC.

The angles might have been found by letting fall a Perpendicular from the Point C, upon the base A B; and reducing it into two Triangles; and then you would have in one triangle the angle at the base, and the angle at the Perpendicular, and the Hypothenusal AC, to find the perpendicular and base; then if you substract the base found, from the first base given, there will remain the base of the other Triangle; then will you have in the second Triangle the base and perpendicular to find the angle at the base; and if you substract the angle at the base from 90 *d.* there will remain the Angle at the Perpendicular; this Angle at the Perpendicular so found, added to the former angle at the Perpendicular, doth give the angle at C.

Note, The former angle at the Perpendicular, was found by ſubſtracting of the angle at A, from 90 d. Then when you have found theſe angles, you may find the ſide B C, having oppoſed ſides and angles, as you ſee in *Caſe* 8.

Caſe 11.

Having the three ſides of an oblique Triangle given to find the Angles.

Fig. 7.

Let the ſide A B be 100 equal parts, and the ſide A C 80, and the ſide C B 64.

For the Laying down this Triangle.

Firſt, Draw a baſe, and make it equal to 100 equal parts; then take off your ſide A C, from the ſame ſcale as you did the baſe; and with that diſtance, ſetting one foot of your Compaſſes in the point A; with the other make a ſmall arch: then take off the length of your ſide B C, and ſetting one foot of your Compaſſes in the point B, with the other interſect the ſaid arch, and where theſe two arches do interſect, is the place of the angles at C; therefore from that place draw B C and A C, and you will have done.

For the finding of either of the Angles.

Before you can find an angle, you must reduce this oblique Triangle into two right angled Triangles: Which is done thus;

First add the side A C, and the side B C together into one sum, and that will be the length of the line A D; then substract the side C B from the side A C, and there will remain a part of the side A C, (*viz.*) the line A E; which is their Difference: Then say,

As the log. of the true base A B,
Is to the log. of the sum of the sides A C, and C B, (*viz.*) A D;
So is their Difference (*viz.*) A E.
To the Log. of part of the base A O.

Which part being substracted from the whole base, upon the middle of the Remainer is the place where the Perpendicular must fall, that will reduce this Triangle into two right angles (*viz.*) the right angled Triangle B D C, and the right angled Triangle A D C; thus:

The length of the base A B is 100.
The part found A O, Substract, 23.
The remainer O B, ——— 77
The half thereof is B D ——— 38;

Which is the place where the Perpendicular muſt fall, and is the baſe of the leſſer Triangle B D G; then if you ſubſtract this leſſer baſe B D, from the whole baſe A B, there will remain the greater baſe A D of the Triangle A D C; then have you the baſe and hypothenuſal in each Triangle, to find an angle; which you find by the Fourth Caſe; then if you add both the angles of the Perpendicular together, their ſum will be the angle at C.

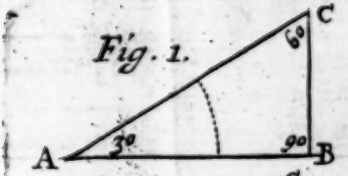
Operation.

As the log. of the ſide B C,
Is to the radius or ſine of 90 *d.*
So is the log of the baſe B D,
To the ſine of the angle at the Perpendicular.
Whoſe Complement to 90 *d.* is the angle at B.

In the greater Triangle.

As the log. of the ſide A C,
Is to the radius or ſine of 90 *d.*
So is the log. of the baſe A D.
To the ſine of the angle at the Perpendicular.
Whoſe Compl. to 90 *d.* is the angle at the baſe A.

The



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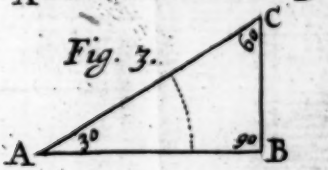

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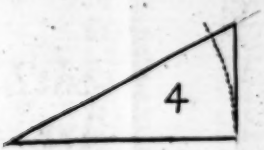

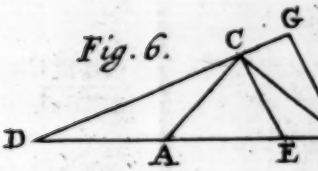

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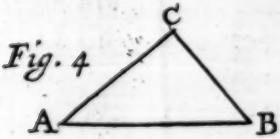
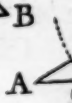


Fig. 4



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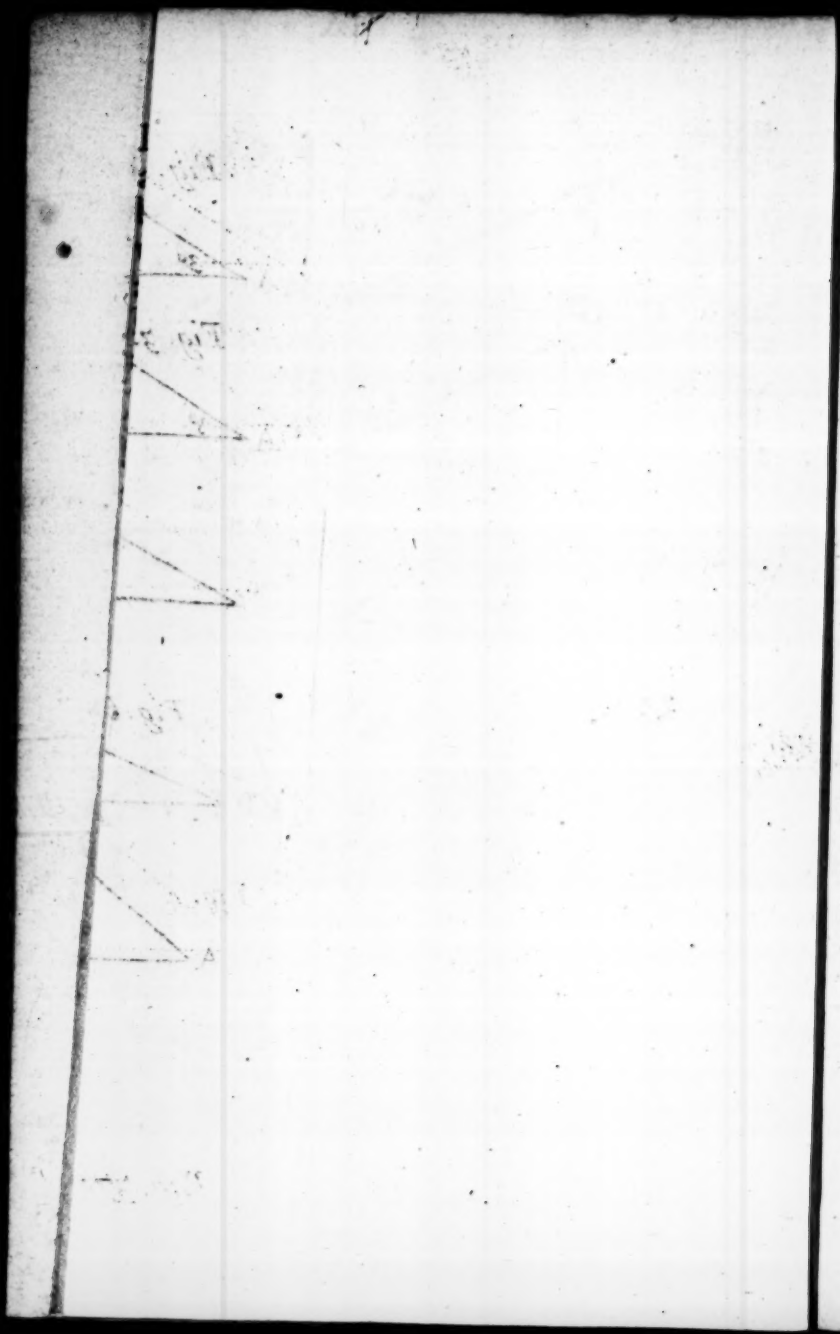


	d.	m.
The angle at the Perpendicular, is	36	59
The angle at the Perpendicular, is	51	15
Their sum is the angle at C,	96	14

CHAP. I.

Containing the Affections of Spherical Triangles, and their Axioms.

1. A Spherical Triangle is that which is described on the Surface of the Globe or Sphere.

2. The sides of a Spherical Triangle are arches of three of the greatest Circles of the Sphere mutually intersecting each other.

3. Those are great Circles which divide the Sphere into two equal parts.

4. Those Circles which cut each other at right angles, one of them doth pass through the Poles of the other, and the contrary.

5. The measure of all Spherical Triangles are upon great Circles of the Sphere described from

from the angular points of their Poles, and ſubtending their angles.

6. The Diſtance of the Poles of two great Circles is equal to the angle comprehended by them; and the contrary.

7. Therefore if you have three ſides of any Spherical Triangle given, there are three angles of another Triangle, that is made by the mutual interſecting of the Circles of which the ſides are formed in the Triangle whoſe ſides are given, that is equal thereunto.

☞ *Note*, only the Complement of the greateſt ſide, or greateſt angle to a Semicircle, muſt be taken in ſuch Converſions.

8. The ſum of the two ſides of any Spherical Triangle are leſs than a whole Circle, *Reg.* 39. 3.

9. The ſum of the three angles of any Spherical Triangle are greater than two right angles, and leſs than ſix, *Reg.* 49. 3.

10. Two angles of any Spherical Triangle are greater than the difference between the third angle and Semicircle; therefore if any ſide be continued, the exterior angle is leſs than the two interior oppoſite ones.

11. In any Spherical Triangle the difference of the ſum of two angles and a whole Circle, is greater than the difference of the third angle and a ſemicircle.

12. A Spherical Triangle is either right or oblique angled.

13. A right angled Spherical Triangle is that which hath one right angle at the leaſt.

14. The Perpendicular and Baſe of a Spherical Triangle that is right angled, are of the ſame affection with their oppoſite angles, *Reg.* 3, 4.

15. In a right angled Spherical Triangle, if either the Perpendicular or Baſe be a Quadrant, the Hypothenuſal ſhall alſo be a Quadrant; but if both of them be a Quadrant, the Hypothenuſal ſhall be leſs than a Quadrant; if of different affection, then greater, and the contrary, *Reg.* 45, 4.

16. In a right angled Spherical Triangle, if either the angle at the Baſe, or the angle at the Perpendicular, be a right angle, the Hypothenuſal ſhall be a Quadrant; but if both of them ſhall be of the ſame affection, it ſhall be leſs; if different, then greater; and on the contrary, *Reg.* 67, 4.

17. In any right angled Spherical Triangle either of the oblique angles is greater, then the complement of the other; but if leſs, then the difference of the ſame complement to a Semicircle.

18. An oblique angled Spherical Triangle is either acute angled, or obtuſe angled: an acute angle is leſs than 90 *d.* and an obtuſe angle is more.

19. An

19. An acute angled Spherical Triangle hath all its angles acute.

20. An obtuſe angled Spherical Triangle hath all its angles obtuſe or mixt (*viz.*) acute and obtuſe.

21. An acute angled ſpherical Triangle each ſide is leſs than a Quadrant, *Reg.* 9. 4.

22. In an oblique angled ſpherical Triangle, if the two acute angles be equal, the ſides oppoſite to them ſhall be leſs than a Quadrant, if obtuſe, greater, *Reg.* 10. 11, & 4.

23. In an oblique angled ſpherical Triangle, if two acute angles be unequal, the ſide oppoſite to the leſſer angle, ſhall be leſs than a Quadrant; but if obtuſe, the ſide oppoſite to the greater angle ſhall be greater than a Quadrant.

24. In every oblique angled ſpherical Triangle, if the angles at the Baſe be of the ſame affection, the Perpendicular drawn from the top of the vertical angle ſhall fall within the Triangle; if different, without. *Reg.* 8. 4.

25. In right angled ſpherical Triangles, there are ſixteen Caſes or Problemes; and twelve in oblique: For the ſolution of which, theſe four following *Axioms* are ſufficient; the three firſt ſerving to reſolve all the *Caſes* of *Right Angled Spherical Triangles*: and ten of the Oblique, they being firſt reduced into two right angles, by letting fall a Perpendicular

cular from the vertical angle; which ſometimes falls without the Triangle, and ſometimes within, as ſhall be ſhewn hereafter.

AXIOM I.

THe Sine of the middle part and Radius being added together, is equal to the Tangents of the Extreams adjacent, and to the Coſines of the Extreams disjunct.

Note, This univerſal Propoſition (the Invention of the Lord *Nepair* Baron of *Merchſton*) will reſolve the ſixteen Caſes of *Right Angled Spherical Triangles*; and will alſo find the firſt Arch of ten Caſes of the *Oblique Spherical Triangles*; as ſhall be cleared by Examples in its proper place.

AXIOM II.

IN all *Right Angled Spherical Triangles*, having the ſame acute Angle at the Baſe, the ſines of the Baſes are directly proportional to the Tangents of their Perpendiculars; and the contrary

AXIOM III.

IN all *Right Angled Spherical Triangles* having the same acute Angle at the Base, the sines of the Hypothenusa's are directly proportional to the sines of their Perpendiculars; and the contrary.

AXIOM IV.

IN all *Spherical Triangles*, whose sides are known, as the Rectangle of the sines of the containing sides is to the Radius, so is the Rectangle of the sines of half the sum of the three sides, and of the difference of the Base there-from, to the Square of the Cosine of half the Angle sought.

From these foregoing Axioms *may be drawn these following Conclusions, which are in number seven.*

1. The Sines of the Angles subtended by the Perpendicular, are directly proportional

to the fines of the Perpendicular ſubtending, and the contrary.

2. If two Perpendiculars ſubtend equal angles, the fines of the Perpendicular are directly proportional to the fines of the Hypothenuſal; and the contrary.

3. The fines of all Anglesare proportional to the fines of their oppoſite ſides; and the contrary.

4. If in an oblique Spherical Triangle a Perpendicular be let fall from the vertical angle to the oppoſite ſide (continued if need be) then the coſines of the angles at the Baſe are directly proportional to the fines of the vertical angles; and the contrary.

5. The coſines of the Segments of the Baſe are directly proportional to the coſines of the ſides that do contain the vertical angle.

6. The fines of the Segments of the Baſe are directly proportional to the Tangents of the angles lying near to the Baſe; and the contrary.

7. The Coſines of the vertical Angles are mutually proportional to the Tangents of the ſides lying near to the vertical angles; and the contrary.

Theſe three laſt Concluſions are ſuppoſed to have Perpendiculars let fall from the vertical Angle to the oppoſite ſide (continued if need be.)

Before

Before my Lord *Nepair's* Universal Proposition can be understood, you must continue the sides of the right angled Spherical Triangle unto Quadrants; so that the Angles may be turned into Side, and the Hypothenusa into Bases and Perpendiculars; and the contrary; by which means the Proportions, as to the Parts of the Triangle given, instead of Sines, do sometimes fall into Co-sines, and sometimes into Co-tangents instead of Tangents; such Parts as do change their Proportion, are noted with their Complements (*viz.*) the Hypothenusa and the Angle at the Base, and the Angle at the Perpendicular: but the Base and the Perpendicular are not so noted.

There are 5 Circular Parts belonging unto a right angled Spherical Triangle. Amongst which the right angled is not accounted as one; therefore the Base and the Perpendicular are supposed to be joyned together.

Any of these Circular Parts may be made the middle part; and then the two Extreams joyning to the middle part, are called Extreams adjacent or conjunct; and the other two that lie remote from the middle Part, are called Extreams disjunct, as in the Triangle A B C, right angled at B: the Angle at the Base A, is noted with Complements, and the Hypothenusa A C is noted with Complements;

ments; and the Angle at the Perpendicular C, is noted with Complements alſo.

If the Baſe A B, be made the middle Part, then is the Angle at the Baſe one of the Extreams adjacent, and the Perpendicular is the other; the Hypothenuſa and the Angle at the Perpendicular are the Extreams disjunct.

That you may the better underſtand what I have written, I have added the two following Tables.

If the Baſe A B *be the middle part,*

Then the Angle at ——A and the Perpendicular B C	are *Extreams Adjacent.*
The Hypothenuſa — A C and the Angle ———C	are *Extreams Disjunct.*

If the Angle A *be a Middle Part,*

Then the Baſe ——— A B And the Hypothenuſa A C	are *Extreams* *Adjacent.*
The Angle at ——— C and the Perpendicular B C	are *Extreams* *Disjunct.*

If the Hypothenuſa A C *be the middle part,*

Then the Angle——— A and the Angle——— C	are *Extreams* *Adjacent.*
The Perpendicular — B C and the Baſe ——— A B	are *Extreams* *Disjunct.*

If

If the Angle C *be the middle part,*

Then the Hypothenuſ. AC and the Perpendic. — BC	are *Extreams* *Adjacent.*
The Angle at ———— A and the Baſe ———— A B	are *Extreams* *Disjunct.*

If the Perpendicular B C *be the middle part*

Then the Baſe ———— A B and the Angle at ———— C	are *Extreams* *Adjacent.*
The Angle at ———— A and the Hypothenuſa A C	are *Extreams* *Disjunct.*

By this foregoing Table you may underſtand when any ſide or angle is made the middle part, and which of the other ſides and angles are Extreams adjacent and disjunct.

And by this following Table you may understand how the Parts noted with Complements, are to change their Terms, sometimes into Co-sines and Co-tangents, and their Co-sines into Sines.

Middle part.	*Extr. Adjac.*	*Extr. disjunct.*
Sine —— A B	Co-tang. A. Tang. B C.	Sine — A C. Sine —— C.
Co-sine — A.	Co-tang. A C. Tang. — A B.	Sine —— C. Co-sine B C.
Co-sine A C.	Co-tang. A. Co-tang. C.	Co-sine A B. Co-sine B C.
Co-sine — C.	Co-tang. A C. Tang. — B C.	Sine —— A. Co-sine A B.
Sine —— B C.	Co-tang. — C. Tang. — A B.	Sine —— A. Sine — A C.

Quadrantal Triangles are those that have one side therein a Quadrant; and yet my Lord *Nepair's* Universal Proposition will resolve all the Parts belonging thereto.

This Quadrantal Triangle is made by the continuing

continuing of the Perpendicular in a Right angled Spherical Triangle to a Quadrant; by which means the Complement of the Perpendicular is one Side thereof, and the Hypothenusa is another Side, and the Quadrantal side is the other.

Note, Whereas in Right angled Spherical Triangles, the Side and Angles lying remote from the right Angle, were noted with their Complements; so in the Quadrantal the angle and sides lying remote from the Quadrantal Sides, are noted with their Complements, and the angles lying next unto the Quadrantal Side; are not noted with their Complem. the Parts thus noted in a Quadrantal Triangle, are called the Circular Parts of a Quadrantal Triangle, of which Parts there are five, the Quadrantal Side not being accounted as one.

The resolving of these Quadrantal Triangles are the same with the Right angled ones; and therefore I shall forbear the rehearsing of them.

CHAP. II.

Containing the Resolution of the Sixteen Cases of Right Angled Spherical Triangles, and consequently the Quadrantal Ones.

Case 1.

The Hypothenusa and Angle at the Base being given, to find the Base A B.

Fig. 1.

LEt the Hypothenusa A C be 50 deg. and the angle at the base A, 23 *d.* 30 *m.*

Here the angle at the base A, is the middle Part, because the Parts given, and Part demanded lie together; likewise the Hypothenusa and angle at the base, are noted with their Complements; therefore you must add the Radius unto the Co-Sine of the middle Part A; and from their Sum substract the Co-tangent of the Extream adjacent A C, and there will remain the Tangent of the other Extream adjacent A B. Thus;

The

The Co-Sine of the middle Part A 23 *d.* 30 *m.*		9962398
The Radius add. B 90. 00		10000000
Their Sum is		19962398
The Co-Tangent of the Extream Adjacent		9923814
Substract A C 50 *d.* 00 *m.*		
The Tangent of the Extream adjacent A B, 47 *d.* 31 *m.*		10038584

Case 2.

The Hypothen. and Angle A. being given, to find the Perpendicular,

Fig. 1.

Let the Hypothenusa A C be 50 *d.* and the Angle at the Base A, be 23 *d.* 30 *m.*

Here the Part demanded is the middle part, because separated from the Parts given by a Side and an Angle; therefore the Parts given are Extreams disjunct: then if you add the Sines of both the Extreams disjunct together, because the Extreams are noted with their Complements, and from that Sum substract the Radius, there will remain the Sine of the middle part: Thus,

The Sine of the Extream A C
50 *d.* 00 *m.* ——— 9884254
The Sine of the Extream, A,
add 23 *d.* 30 *m.* ——— 9600700

Their Sum is ——— 19484954
The Radius substract, B, 90. 00. 10000000

The Sine of the middle part
B C, 17 47, ——— 9484954
Which is the Perpend. desired, ———

Case 3.

The Hypothenusa and Angle at the Base being given, to find the Angle at the Perpendicular.

Fig. 1.

Let the Hypothenusa A C, be 50 *d.* and the Angle at the Base A, be 23 *d.* 34 *m.*

Here the Hypothenusa is the middle part, because the parts given and part demanded lie together, and they are all of them noted with their Complements; therefore add the Radius to the Co-sine of the middle part, and from their Sum substract the Co-tangent of the Extream given, and there will remain the Co-tangent of the Extream demanded. Thus,

The

The Co-sine of the middle part
A C, 50 *d.* 00 *m.* ——— 9808067
The Radius 90 *d.* 00 ——— 1000000

Their Sum is ——— 19808067
The Cotangent of the Angle A,
substract. 23 *d.* 30 *m.* ——— 10361698

The Co-tangent of the Angle C,
74 *d.* 23. *m.* ——— 9446369
Which is the Angle at C desired, ———

Case 4.

The Base and Angle at the Base being given, to find the Hypothenusa:

Let the Base A B be 47 *d.* 32 *m.* and the Angle at the Base be 23 *d.* 30 *m.*

In this Case the Angle at the Base A, is the middle part, and the Base and the Hypothenusa is the Extreams adjacent; the middle part and one Extream being noted with their Complement, if to the Co-sine of the Middle Part you add the Radius, and from that Sum substract the Tangent of the given Extream, there will remain the Co-tangent of the other Extream: Thus;

The

The Co-sine of the midddle Part A,
23 *d.* 30 *m.* ——— 9962398
The Radius add. B 90 *d.* 00 *m.* — 10000000

Their Sum is ——— 19962398
The Tangent of the Extream A B,
Subſtr. 47 *d.* 32 *m.* ——— 10038455

The Co-tangent of the Extream A C,
50 *d.* 00 *m.* ——— 9923943

Which is the length of the Hypothenuſal required.

Caſe 5.

The Baſe and Angle at the Baſe being given, to find the Perpendicular.

Fig. 1.

Let the Baſe A B, be 47 *d.* 32 *m.* and the Angle at the Baſe A, be 23 *d.* 30 *m.*

Here the Baſe is the middle part, becauſe the parts given, and the parts demanded lie together, and by reaſon that the Angle at the Baſe is noted, with its Complement, the Rule goes thus;

If from the Sine of the middle part and Radius added together, you ſubſtract the Co-

tangent

tangent of one of the Extreams, there will remain the Tangent of the other; Thus:

The Sine of the middle Part A B, 47 *d.* 32 *m.*	9867862
The Radius add B, 90 *d.* 00 *m.*	10000000
Their Sum	19867862
The Co-tangent of the Angle A Subſtract. 23 *d.* 30 *m.*	10361698
The Tangent of the Perpendicular BC, 17 *d.* 47 *m.*	9506164

which was required.

Caſe 6.

The Baſe and the Angle at the Baſe being given, to find the Angle at the Perpendicular;

Fig. 1.

Let the Baſe A B, be 47 *d.* 32 *m.* and the Angle at the Baſe A, be 23 *d.* 30 *m.*
Here the Angle at the Perpend. is the middle part, and both the Parts given are Extreams disjunct; and becauſe the Angle at the Baſe is noted with Complements, the Rule goes thus;

If

If to the Co-sine of one Extream disjunct you add the Sine of the other, and from their Sum substract the Radius, there will remain the Co-sine of the middle Part (because the middle part is noted with Complement; thus;

The Co-sine of the Extream A B,
47 *d.* 32 *m.* ——————— 9829407
The Sine of the Extream add. A,
23 *d.* 30 *m.* ——————— 9600700

Their Sum is ——————— 19430107
The Radius subst. B, 90 *d.* 00 *m.* 1000000

The Co-sine of the middle part C,
74 *d.* 23 *m.* ——————— 9430107

Which is the Angle of the Perpendicular required.

Case 7.

The Perpendicular and Angle at the Base being given, to find the Base;

Fig. 1.

Let the Angle at the Base A, be 23 *d.* 30 *m.* and the Perpendicular B C, 17 *d.* 47 *m.*

Here the Base demanded is the middle part, and the Extreams given are adjacent, because

one

one of the Extreams given is noted with Complement, you muſt add the Tangent of the given Extream B C, to the Co-tangent of the other given Extream A; and from their Sum ſubſtract the Radius, and there will remain the Sine of the middle part demanded; Thus,

The Tangent of the Extream B C, 17 *d.* 47 *m.*	9506159
The Co-tangent of the Extream Add. A, 23 *d.* 30 *m.*	10361698
Their Sum is	19867857
The Radius ſubſtr. B, 90 *d.* 00 *m.*	10000000
The Sine of the middle part A B, 47 *d.* 32 *m.*	9867857

Which is the part required.

Caſe 8.

The Angle at the Baſe, and the Perpendicular being given, to find the Angle at the Perpendicular.

Fig. 1.

Let the Angle at the Baſe A, be 23 *d.* 30 *m.* and the Perpendicular B C, be 17 *d.* 47 *m.*

In this caſe the angle at the baſe A, is the middle part, and the other two parts are Extreams disjunct:

disjunct: Here the middle part and one of the Extreams is noted with Complement.

The Rule to find the Extreams, is thus;

To the co-sine of the middle part add the Radius, and from their Sum subſtract the co-sine of the given Extream, and there doth remain the Sine of the other Extream; thus:

The co-sine of the middle A, 23 *d.* 30 *m.*	9962398
The Radius add. B 90 *d.* 00 *m.*	10000000
Their Sum is	19962398
The Co-sine of the given Extream subſtr. B C, 17 *d.* 47 *m.*	9968737
There remains the Sine of the An- C 74 *d.* 23 *m.*	9983661

Which is the Angle required.

Caſe 9.

The Angle at the Baſe and the Perpendicular being given, to find the Hypothenuſa.

Fig. 1.

Let the Angle at the Baſe A, be 23 *d.* 30 *m.* and the Perpendicular B C, be 17 *d.* 47 *m.*

In this caſe the Perpendicular is the middle part,

part, and the part demanded, and the Angle at the Baſe are Extreams disjunct, and are both noted with their complements; therefore if you add the Radius unto the Sine of the middle part, and from their Sum ſubſtract the Sine of one of the Extreams given, there will remain the ſine of the Extream demanded: Thus;

The Sine of the middle part B C, 17 *d.* 47 *m.*	9484895
Thereto add the Radius B, 90 *d.* 00 *m.*	10000000
From their Sum, which is	19484895
Subſtract the Sine of the Extream A, 23 *d.* 30 *m.*	9600700
There remains the Sine of the Extream A C, 50 *d.*	9884195
The Hypoth. required.	

Caſe 10.

The Hypothenuſa and Baſe being given, to find the Angle at the Baſe.

Fig. 1.

Let the Hypothenuſa A C be 50 *d.* and the Baſe A B, be 47 *d.* 32 *m.*

In

In this Case the Angle at the Base is the middle part, and the base and hypothenusa are Extreams adjacent; one Extream and the middle part are noted with their complements: Now if to the Tangent of the Extream A B, you add the co-tangent of the Extream A C, and from their Sum you substract the Radius, there will remain the co-sine of the middle part: thus;

Unto the tangent of the base A B, 47 *d.* 32 *m.* ——— 10038555
Add the co-tangent of the Hypothenusa A C 50 *d.* 00 *m.* ——— 9923914

And from their Sum, which is — 19962369
substract the radius B, 90 *d.* 00 *m.* ——— 10000000

There will remain the co-sine of the Angle A, which is 23 *d.* 30 *m.* ——— 9962269
Which is the part sought.

Case 11.

The Hypothenusa and Base being given to find the Perpendicular.

Fig. 1.

Let the Base A B be 47 *d.* 32 *m.* and the Hypothenusa A C be 50 *d.* In

In this case the Hypothenusa is the middle part, the base and perpendicular are Extreams disjunct; and there being none but the middle part noted with complement; therefore add the co-sine of the middle part unto the Radius, and from their sum substract the co-sine of the given Extream, and there will remain the co-sine of the Extream demanded:

Thus:

To the co-sine of the middle part
A C, 50 *d.* 00 *m.* ——— 9808067
Add the Radius B, 90 *d.* 00 *m.* — 10000000

And from their Sum, which is — 19808067
Substract the co-sine of the base
A B 47 *d.* 32 *m.* ——— 9829407

There will remain the co-sine of the
Perpendicular B C, 17 *d.* 47 *m.* — 9978660
Which is the part demanded.

Case 12.

The hypothenusa and base being given, to find the angle at the perpendicular.

Fig. 1.

Let the Hypothenusa A C be 50 *d.* and the base A B be 47 *d.* 32 *m.*

In this case the base is the middle part, because it doth lie remote from the other two, and because the hypothesusal and angle at the perpendicular, are noted with complements, you must add the radius to the sine of the middle part, and from their sum substract the sine of the Extream A C, and there will remain the sine of the angle at the Perpendicular: Thus;

The sine of the middle part A B, 47 *d.* 32 *m.*
The Radius add
From their Sum,
Substract the sine of the Hypothenusal A C, 50 *d.* 00 *m.*
There doth remain the sine of the angle at the perpendicular C.

Case 13.

The Hypothenusal and Perpendicular being given, to find the angle at the base.

Fig. 1.

Let the hypothenusal be 50 *d.* and the perpendicular be 17 *d.* 47 *m.*

The Perpendicular is here the middle part, and the angle at the base and hypothenusal, are Extreams disjunct; then to the sine of the

the middle part B C, add the Radius, and from their ſum ſubſtract the ſine of the Hypothenuſal A C, and there will remain the ſine of the angle at the baſe A.

Thus;

To the ſine of the Perpend. B C 17 *d.* 47 *m.*
Add the Radius
From their Sum,
Subſtract the ſine of A C, 50 *d.* 00 *m.*
There will remain the ſine of the angle at the baſe, A.

Caſe 14.

The Hypothenuſal and Perpendicular being given, to find the baſe;

Fig. 1.

Let the hypothenuſal be A C, 50 *d.* 00 *m.*
The perpendicular ——— B, 17 *d.* 47 *m.*

Here the hypothenuſal is the middle part, becauſe it doth lie remote from the other two; and becauſe the middle part A C is noted with complements, it muſt be a co-ſine; then to the coſine of the middle part, add the Radius; and from their Sum ſubſtract the coſine of the Extream given B C, and there will remain the co-ſine of the Extream demanded, A B. Thus;

In this case the base is the middle part, because it doth lie remote from the other two; and because the hypothesusal and angle at the perpendicular, are noted with complements, you must add the radius to the sine of the middle part, and from their sum substract the sine of the Extream A C, and there will remain the sine of the angle at the Perpendicular: Thus;

The sine of the middle part A B, 47 *d.* 32 *m.*
The Radius add
From their Sum,
Substract the sine of the Hypothenusal A C,
50 *d.* 00 *m.*
There doth remain the sine of the angle at the perpendicular C.

Case 13.

The Hypothenusal and Perpendicular being given, to find the angle at the base.

Fig. 1.

Let the hypothenusal be 50 *d.* and the perpendicular be 17 *d.* 47 *m.*

The Perpendicular is here the middle part, and the angle at the base and hypothenusal, are Extreams disjunct; then to the sine of the

the middle part B C, add the Radius, and from their ſum ſubſtract the ſine of the Hypothenuſal A C, and there will remain the ſine of the angle at the baſe A. Thus;

To the ſine of the Perpend. B C 17 *d.* 47 *m.*
Add the Radius
From their Sum,
Subſtract the ſine of A C, 50 *d.* 00 *m.*
There will remain the ſine of the angle at the baſe, A.

Caſe 14.

The Hypothenuſal and Perpendicular being given, to find the baſe;

Fig. 1.

Let the hypothenuſal be A C, 50 *d.* 00 *m.*
The perpendicular ——— B, 17 *d.* 47 *m.*

Here the hypothenuſal is the middle part, becauſe it doth lie remote from the other two; and becauſe the middle part A C is noted with complements, it muſt be a co-ſine; then to the coſine of the middle part, add the Radius; and from their Sum ſubſtract the coſine of the Extream given B C, and there will remain the co-ſine of the Extream demanded, A B. Thus;

The co-ſine of the hypothenuſal A C 50 *d.* 00 *m.*
The Radius add.
From their Sum,
Subſtract the co-ſine of the Perpendicular B C, 17 *d.* 47 *m.*
There will remain the co-ſine of the baſe A B.

Caſe 15.

The Hypothenuſal and Perpendicular being given, to find the Angle at the Perpendicular.

Fig. 1.

Let the Hypothenuſal be 50 *d.* 00 *m.*
The Perpendicular ——— 17 *d.* 47 *m.*

Here the angle at the Perpendicular is the middle part, and the two parts given are the Extreams conjoyned; therefore to the co-tangent of the Hypothenuſal add the Tangent of the Perpendicular; from their Sum ſubſtract the Radius, and there will remain the co-ſine of the middle part.

Thus;

The Co-tangent of the Hypothenuſal A C, 50 *d.* 00 *m.*
The Tangent of the Perpendicular, B C, 17 *d.* 47 *m.* add

From

From their Sum,
Subſtract the Radius,
There will remain the Co-ſine of the middle part, the angle at the Perpendicular.

Caſe 16.

The Angle at the Baſe and the Angle at the Perpendicular being given, to find the Hypothenuſa.

Fig. 1.

Let the angle at the baſe be *A*, 30 deg.
Let the angle at the perpendicular be C, 60 deg.

In this caſe the hypothenuſal is the middle part, and the angles are Extreams conjunct; then if you add the co-tangents of both the angles together, and from their Sum, ſubſtract the Radius, there will remain the co-ſine of the middle part: Thus;

The co-tangent of the angle at A 30 *d.*
Add
The co-tangent of the angle at C, 30 *d.*
Their Sum will be
From which ſubſtract the radius 90 *d.* 00 *m.*
There will remain the co-ſine of the Hypothenuſal.

Thus have I run over all the Sixteen Cases of Right Angled Spherical Triangles, according to my Lord *Napier's Universal Proposition*; but those that please to turn them into *Analogies*, must note, that, that Number which is to be substracted from the Sum of the other two in all the Work afore-going, is the first Term of the Proportion; and either of the two Numbers that are added into one Sum, may be made the second Term; and the remaining Number must be the third Term; and the fourth will be the Number sought, let it be either Sine, Tangent or Secant: And that the young Student may see how a *Right Angled Spherical Triangle* may be varied, I have presented him with the Sixteen Cases of *Right-Angled Spherical Triangles*, in the following Chapter, as I find them Translated from *Clavius* his *Astrolabe*.

CHAP.

CHAP. III.

The Sixteen Cases of Right Angled Spherical Triangles, *Translated from* Clavius, *De* Astrolabio.

Case 1.

To find an Angle.

Given the Hypothenusal and Side opposite to the Angle sought.

As the Sine of the Hypothenusal,
To the Radius;
So the Sine of the given side,
To the sine of the Angle sought.

Or,

As the Radius
To the sine of the Hypothenusal;
So the Co-secant of the side,
To the Co-secant of the Angle.

As the Radius
To the ſine of the Side;
So the Co-ſecant of the Hypothenuſal
To the ſine of the Angle.

As the Co-ſecant of the ſide
To the Radius;
So the Co-Secant of the Hypothenuſal,
To the Co-Secant of the Angle.

As the Co-ſecant of the Hypothenuſal
To the Radius;
So the Co-ſecant of the ſide,
To the Co-ſecant of the Angle.

As the ſine of the ſide
To the Radius:
So the ſine of the Hypothenuſal
To the Co-ſecant of the Angle.

The Angle found will be acute if the Side given be leſs than a Quadrant; obtuſe if greater.

Caſe

Case 2.

To find an Angle.

Given the Hypothenuſal and ſide adjacent to the Angle.

As the Radius,
To the Co-tangent Hypothenuſal:
So the Tangent of the ſide,
To the Co-ſine of the Angle.

As the Tangent of the Hypothenuſal,
To the Radius:
So the Tangent of the ſide,
To the Co-ſine of the Angle.

As the Co-tangent of the ſide,
To the Radius:
So the Co-tangent of the Hypothenuſal,
To the Coſine of the Angle.

As the Radius,
To the Co-tangent of the ſide:
So the Tangent of the Hypothenuſal,
To the Secant of the Angle.

As the Co-tangent of the Hypothenuſal
To the Radius,

So

So the Co-tangent of the ſide,
To the Secant of the Angle.

As the Tangent of the ſide,
To the Radius:
So the Tangent of the Hypothenuſa,
To the Secant of the Angle.

The Angle found will be Acute, if both the Hypothenuſal and the given ſide be greater or leſs than a Quadrant; but Obtuſe, if one of them be greater, and the other leſs.

Caſe 3.

To find an Angle.

Given the Hypothenuſal, and either of the Oblique Angles.

As the Radius
To the Co-ſine of the Hypothenuſa:
So the Tangent of the given Angle,
To the Co-tangent of the ſought Angle.

As the Co-tangent of the given Angle
To the Radius;
So the Co-ſine of the Hypothenuſal,
To the Co-tangent of the ſought Angle.

As

As the Co-ſine of the Hypothenuſal
To the Radius;
So the Co-tangent of the given Angle,
To the Tangent of the Angle ſought.

As the Radius
To the Secant of the Hypothenuſal;
So the Co-tangent of the given Angle,
To the Tangent of the Angle ſought.

As the Secant of the Hypothenuſal
To the Radius;
So the Tangent of the given Angle,
To the Co-tangent of the Angle ſought.

As the Tangent of the given Angle
To the Radius;
So the Secant of the Hypothenuſal
To the Tangent of the Angle ſought.

The Angle found will be acute, if the Hypothenuſal be leſs than a Quadrant, and the given Angle acute; or if the Hypothenuſal be greater than a Quadrant, and the given Angle obtuſe; and the ſaid Angle will be obtuſe, if the Hypothenuſal be leſs than a Quadrant, and the given Angle obtuſe; or if the Hypothenuſal be greater than a Quadrant, and the given Angle acute.

Caſe

Case 4.

To find an Angle.

Given the Side oppoſite to the Angle ſought, and the other Oblique Angle.

As the Radius
To the ſine of the Angle given;
So the Co-ſine of the given ſide,
To the Co-ſine of the Angle ſought.

As the Radius
To the Co-ſecant of the given angle;
So is the Secant of the ſide given
To the Secant of the angle ſought.

As the ſine of the given angle
To the Radius;
So the ſecant of the given ſide
To the Secant of the angle.

As the Co-ſine of the given ſide
To the Radius,
So the Co-ſecant of the given Angle
To the Secant of the Angle ſought.

As the Co-ſecant of the given angle
To the Radius;
So the Co-ſine of the given ſide
To the Co-ſine of the angle ſought.

As the Secant of the given ſide
To the Radius;
So the ſine of the given angle
To the Co-ſine of the angle ſought.

The Angle found will be acute, if the ſide given be leſs than a Quadrant; obtuſe, if greater.

Caſe 5.

To find an Angle.

Given a Side adjacent to the Angle ſought, and the other oblique Angle, if it be foreknown whether the Angle ſought be acute or obtuſe, or whether the Baſe or other Side not given, be greater or leſſer than a Quadrant.

As the Co-ſine of the given ſide
To the Radius;
So the Co-ſine of the given angle
To the ſine of the angle ſought.

As

As

As the Radius
To the Secant of the given ſide,
So the Co-ſine of the given angle
To the Sine of the angle ſought.

As the Radius
To the Co-ſine of the given angle;
So the Secant of the given ſide
To the Co-ſecant of the ſought angle.

As the Co-ſine of the given angle
To the Radius,
So the Co-ſine of the given ſide
To the Co-ſecant of the angle ſought.

As the Secant of the given ſide
To the Radius,
So the Secant of the given angle
To the Co-ſecant of the angle ſought.

As the Secant of the given angle
To the Radius,
So is the Secant of the given ſide
To the ſine of the angle ſought.

The Angle found will be acute, if the ſide not given be leſs than a Quadrant, obtuſe, if greater.

In

In like manner, if the Hypothenuſal be leſs than a Quadrant, and the given Angle acute; or if the Hypothenuſal be greater than a Quadrant, and the Angle given obtuſe, the Angle found will be acute.

But if the Hypothenuſal be leſs than a Quadrant, and the given Angle obtuſe, or if the Hypothenuſal be greater than a Quadrant, and the given Angle acute, the Angle found will be obtuſe.

Caſe 6.

To find an Angle.

Given both the Leggs.

As the Radius
To the ſine of the ſide adjacent to the ſought angle,
So the Co-tangent of the ſide oppoſite to the Angle ſought,
To the Co-tangent of the angle ſought.

As the ſine of the ſide adjacent to the ſought Angle
To the Radius,
So the tangent of the ſide oppoſite to the ſought Angle,
To the tangent of the angle ſought.

As

As the tangent of the ſide oppoſite to the ſought Angle
To the Radius,
So the ſine of the ſide adjacent to the ſought Angle,
To the Co-tangent of the angle ſought.

As the Radius
To the Co-ſecant of the ſide adjacent to the ſought augle,
So the tangent of the ſide oppoſite thereto,
To the tangent of the angle ſought.

As the Co-ſecant of the ſide adjacent to the angle ſought
To the Radius,
So is the Co-tangent of the ſide oppoſite to the angle ſought,
To the Co-tangent of the ſought angle.

As the Co-tangent of the ſide oppoſite to the ſought angle,
To the Radius,
So the Co-ſecant of the ſide adjacent to the angle ſought,
To the tangent of the angle ſought.

The Angle found will be acute, if the ſide oppoſite to the Angle ſought be leſs than a Quadrant; but obtuſe, if greater.

Caſe

Case 7.

To find a Side or Legg.

Given the Hypothenuſa and the other Legg.

As the Co-ſine of the given ſide
To the Radius,
So the Coſine of the Hypothenuſal
To the Co-ſine of the ſide ſought.

As the Radius
To the Secant of the ſide,
So the Co-ſine of the Hypothenuſal
To the Co-ſine of the ſide ſought.

As the Co-ſine Hypothenuſal
To the Radius,
So the Co-ſine of the given ſide,
To the Secant of the ſide ſought.

As the Radius
To the Secant of the Hypothenuſal,
So the Co-ſine of the given ſide
To the Secant of the ſide ſought.

As the Secant of the Hypothenuſal
To the Radius
So the Secant of the given ſide
To the Co-ſine of the ſide ſought.

As the Secant of the given ſide,
To the Radius;

So the Secant of the Hypothenusal,
To the Secant of the side sought.

The side sought will be less than the Quadrant, if both the Hypothenusal and given side be less than Quadrants; but greater than a Quadrant, if either the Hypothenusal be greater, and the given side less; or if the Hypothenusal be less, and the given side greater.

Case 8.

To find a Side.

Given the Hypothenusal and an Angle opposite to the Side sought.

As the Radius
To the sine of the Hypothenusal,
So the sine of the given angle,
To the sine of the side sought.

As the Radius
To the Co-secant of the Hypothenusal,
So the Co-secant of the given angle,
To the Co-secant of the side sought.

As the sine of the Hypothenusal
To the Radius,

So

So the Co-ſecant of the given angle
To the Co-ſecant of the ſide ſought.

As the Co-ſecant of the given angle
To the Radius,
So the ſine of the Hypothenuſal
To the ſine of the ſide ſought.

As the ſine of the given angle
To the Radius,
So the Co-ſecant of the Hypothenuſal,
So the Co-ſecant of the ſide ſought.

The Side found will be leſs than a Quadrant, if the Angle oppoſite thereto be acute; but greater, if obtuſe.

Caſe 9.

To find a Side.

Given the Hypothenuſal and Angle adjacent to the Side ſought.

As the Radius
To the Co-ſine of the given angle,
So the Tangent of the Hypotheuuſal,
To the Tangent of the ſide ſought.

As

As the Co-ſine of the given Angle,
To the Radius,
So the Co-tangent of the Hypothenuſal
To the Co-tangent of the ſide ſought.

As the Co-tangent of the Hypothenuſal
To the Radius,
So the Co-ſine of the given angle,
To the tangent of the ſide ſought.

As the Radius
To the Secant of the angle,
So the Co-tangent of the Hypothenuſal
To the Co-tangent of the Side ſought.

As the Secant of the given angle,
To the Radius,
So the tangent of the Hypothenuſal,
To the tangent of the Side ſought.

As the tangent of the Hypothenuſal
To the Radius,
So the Secant of the given Angle
To the Co-tangent of the ſide ſought.

The Side ſought will be leſs than a Quadrant, if the Hypothenuſal be leſs than a Quadrant; and the given Angle acute; Or if the Hypothenuſal be greater than a Quadrant, and the given Angle obtuſe; but it will be greater than

than a Quadrant, if the Hypothenuſal be leſs than a Quadrant, and the given Angle obtuſe: Or if the Hypothenuſal be greater than a Quadrant, and the given Angle acute.

Caſe 10.

To find a Side.

Given a Side, and an Angle adjacent to the ſought Side.

Provided it be foreknown whether the ſide ſought be greater or leſs than a Quadrant, or whether the other Angle not given, be acute or obtuſe; or finally whether the Hypothenuſal be greater or leſs than a Quadrant.

As the Radius
To the Co-tangent of the given Angle,
So the tangent of the given ſide
To the ſine of the ſide ſought.

As the Tangent of the given Angle
To the Radius,
So the tangent of the given ſide,
To the ſine of the ſide ſought.

As the Co-tangent of the given ſide,
To the Radius,

So the Co-tangent of the given Angle,
To the ſine of the ſide ſought.

As the Radius
To the Co-tangent of the given ſide,
So the tangent of the given Angle
To the Co-ſecant of the ſide ſought.

As the Tangent of the Side given,
To the Radius,
So the tangent of the given Angle
To the Co-ſecant of the Side ſought.

As the Co-tangent of the given Angle,
To the Radius,
So the Co-tangent of the given Side
To the Co-ſecant of the Side ſought.

The Side found will be leſs than a Quadrant, if the Angle oppoſite thereto, and not given, be acute; but greater, if it be obtuſe: In like manner, it will be leſs, if the Hypothenuſal be leſs than a Quadrant, and the Side given, alſo leſs than a Quadrant.

Or, if the Hypothenuſal be leſs than a Quadrant, and the given ſide greater, the ſide found will be greater than a Quadrant:

Laſtly, If both the Hypothenuſal and the ſide given be greater than a Quadrant; the ſide found will be leſs than a Quadrant; but greater

er, if the Hypothenusal be greater than a Quadrant, and the given side less.

Case 11.

To find a Side.

Given a Side, and an Angle oppoſite to the Side ſought.

As the Radius
To the ſine of the given ſide,
So the Tangent of the given Angle
To the Tangent of the ſide ſought.

As the ſine of the given ſide
To the Radius,
So the Co-tangent of the given Angle,
To the Co-tangent of the ſide ſought.

As the Co-tangent of the given Angle
To the Radius;
So the ſine of the given ſide,
To the tangent of the ſine ſought.

As the Tangent of the given Angle
To the Radius;
So the Co-Secant of the given Side
To the Co-Tangent of the ſide ſought.

As the Radius
To the Co-secant of the given Side,
So the Co-Tangent of the given Angle,
To the Co-tangent of the Side sought.

As the Co-secant of the given side,
To the Radius;
So the Tangent of the given angle
To the Tangent of the side sought.

The Side found will be less than a Quadrant, if the given Angle opposite thereto, be acute, but greater, if obtuse.

Case 12.

To find a Side.

Given both the Oblique Angles.

As the Sine of the angle adjacent to the Side sought,
To the Radius,
So the Co-Sine of the Angle opposite to the side sought,
To the Co-sine of the Side sought.

As

As the Radius
To the Secant of the angle oppoſite to the ſide ſought
So the Sine of the angle adjacent to the Side ſought.
To the ſecant of the Side ſought.

As the Radius
To the Co-Secant of the angle adjacent to the Side ſought,
So the Co-ſine of the Angle oppoſite to the Side ſought,
To the Co-Sine of the Side ſought.

As the Co-ſine of the angle oppoſite to the Side ſought,
To the Radius,
So the ſine of the angle adjacent to the Side ſought,
To the Secant of the Side ſought.

As the Secant of the angle oppoſite to the Side ſought,
To the Radius,
So the Co-ſecant of the angle adjacent to the Side ſought,
To the Coſine of the Side ſought.

As the Co-ſecant of the angle adjacent to the Side ſought
To the Radius, So

So is the Secant of the angle oppoſite to the ſide ſought,
To the Secant of the Side ſought.

The Side found will be leſs than a Quadrant, if the given Angle oppoſite thereto be acute, but greater, if obtuſe.

Caſe 13.

To find the Hypothenuſal.

Given a Side and an Angle adjacent thereto.

As the Radius
To the Co-ſine of the Angle given,
So the Co-tangent of the given ſide,
To the Co-tangent of the Hypothenuſal.

As the Co-ſine of the given angle
To the Radius,
So the tangent of the given ſide,
To the tangent of the Hypothenuſal.

As the Tangent of the given ſide
To the Radius:
So the Co-ſine of the given Angle,
To the Co-tangent of the Hypothenuſal.

As

As the Radius
To the Secant of the given angle;
So the Tangent of the given ſide
To the Co-tangent of the Hypthenuſal.

As the Secant of the given Angle
To the Radius;
So the Co-tangent of the given ſide
To the Co-tang. of the Hypoth.

As the Co-tangent of the given ſide,
To the Radius,
So the Secant of the given angle
To the tangent of the Hypothenuſal.

The Hypothenuſal found will be be leſs than a Quadrant, if the given ſide be leſs than a Quadrant, and the Angle given adjacent thereto, acute; As alſo if the given ſide be greater than a Quadrant, and the given Angle adjacent thereto obtuſe:

But it will be greater than a Quadrant, if the given ſide be greater than a Quadrant, and the given Angle adjacent thereto acute; As alſo when the given ſide is leſs than a Quadrant, and the given Angle obtuſe.

CASE

Case 14.

To find the Hypothenuſal.

Given a Side, and an Angle oppoſite thereto.

If it be fore-known whether the Hypothenuſal be greater or leſs than a Quadrant, or whether the other Angle not given, be acute or obtuſe; or laſtly, whether the other ſide not given, be greater or leſs than a Quadrant.

As the ſine of the given angle
To the Radius,
So the ſine of the given ſide
To the ſine of the Hypothenuſal.

As the Radius,
To the Co-ſecant of the angle given,
So the Sine of the given ſide,
To the Sine of the Hypothenuſal:

As the Radius,
To the Co ſecant of the given ſide:
So the ſine of the given angle
To the Co-ſecant of the Hypothenuſal.

As

As the Sine of the given ſide
To the Radius,
So the Sine of the given angle
To the Co-ſecant of the Hypothenuſal.

As the Co-ſecant of the given ſide
To the Radius;
So the Co-ſecant of the given angle,
To the ſine of the Hypothenuſal.

As the Co-ſecant of the given angle,
To the Radius;
So the Co-Secant of the given ſide,
To the Co-Secant of the Hypothenuſal.

The Hypothenuſal found will be leſs than a Quadrant, if both the oblique Angles be acute or obtuſe; or if both the ſides be greater or leſs than Quadrants.

It will alſo be greater than a Quadrant, if one of the oblique Angles be acute, and the other obtuſe; or if one of the ſides be leſs, and the other greater than a Quadrant.

Caſe

Caſe 15.

To find the Hypothennſal.

Given both Sides, diſtinguiſhed by the Names of Firſt and Second.

As the Radius
To the Co-ſine of the firſt ſide,
So the Co-ſine of the ſecond ſide,
To the Co-ſine of the Hypothenuſal.

As the Radius
To the ſecant of the firſt ſide,
So the ſecant of the ſecond ſide
To the ſecant of the Hypothenuſal.

As the Secant of the firſt ſide
To the Radius;
So the Co-ſine of the ſecond ſide
To the Co-ſine of the Hypothenuſal.

As the ſecant of the ſecond ſide
To the Radius;
So the Co-ſine of the firſt ſide,
To the Coſine of the Hypothenuſal.

As the Co-ſine of the firſt ſide
To the Radius;
So the ſecant of the ſecond ſide
To the ſecant of the Hypothenuſal.

As the Co-ſine of the ſecond ſide,
To the Radius;
So the ſecant of the firſt ſide,
To the ſecant of the Hypothenuſal.

The Hypothenuſal found will be leſs than a Quadrant, if both the ſides are leſs or greater; but otherwiſe, it will be greater, if one be leſs, and the other greater.

Caſe 16.

To find the Hypothenuſal.

Given both the Oblique Angles, diſtinguiſh'd by the Names of the firſt and Second.

As the Radius
To the Co-tangent of the firſt Angle,
So the Co-tangent of the ſecond Angle
To the Co-ſine of the Hypothenuſal.

As the Tangent of the firſt Angle,
To the Radius:

So

So the Co-tangent of the ſecond Angle,
To the Co-ſine of the Hypothenuſal.

As the Tangent of the ſecond Angle,
To the Radius,
So the Co-tangent of the firſt Angle
To the Co-ſine of the Hypothenuſal.

As the Radius
To the Tangent of the ſecond Angle,
So the Tangent of the firſt Angle,
To the Secant of the Hypothenuſal.

As the Co-tangent of the ſecond angle
To the Radius,
So the Tangent of the firſt angle,
To the ſecant of the Hypothenuſal.

As the Co-tangent of the firſt Angle
To the Radius:
So the Tangent of the ſecond Angle
To the ſecant of the Hypothenuſal.

The Hypothenuſal found will be leſs than a Quadrant, if both the oblique Angle be acute or obtuſe, but greater if one of them be acute, and the other obtuſe.

CHAP.

CHAP. IV.

Of Oblique Spherical Triangles.

Case 1.

Two Angles and a Side opposite to one of them, being given to find the Side opposite to the other, if it be known, whether the Side sought be more or less than a Quadrant.

LEt the Angle at A, be 40 deg.
And the Angle at B, be 30 deg.
And the Side A C, be 50 deg.
And let the Side B C, be required:
The Proportion will run thus;
As the Sine of the Angle B, 30 deg.

Co. Ar.	0301030
Is to the sine of the side A C, 50 *d.*	9884254
So is the sine of the Angle at A, 40 *d.*	9808067
To the sine of the side B C, 80 *d.*	9993351

Note, If the ſum of the given Angles exceed 180 *d.* the oppoſite Triangle muſt be reſolved.

☞ *Note*, The two ſides together with their oppoſite Angles thus found, the third ſide may be obtained by this following Propoſition; Thus:

As the ſine of half the difference of the given Angles,
Is to the ſine of half the ſum of the given Angles,
So is the Tangent of half the difference of the ſides,
To the Tangent of half the other ſide required.

Caſe 2.

Two Sides with the Angle oppoſite to one of them given, to find the Angle oppoſite to the other ſide, if it be known, whether it be Acute or obtuſe.

Fig. 2.

Let the Side A B be 64 deg. and the ſide B C be 43 deg. and the Angle A. be 40 deg. and let the Angle C be required:

The

The Proportion will run thus;

As the Sine of the ſide oppoſite to the Angle given B C, 43 *d. Co. Ar.* ——— 0176627
Is to the ſine of the given Angle A,
40 *d.* ——— 9808067
So is the ſine of the ſide oppoſite to the Angle demanded A B, 64 *d.* ——— 9953660

To the ſine of the Angle enquired C,
which is 60 *d.* 11 *m.* ——— 9938354

This 60 *d.* 11 *m.* being ſubſtracted from 180 *d.* leaves the Angle C, required.

The two Angles together with their oppoſite ſides thus found, the third Angle may be had after the like manner as the third Side was in the laſt Caſe. Thus:

As the ſine of half the difference of the Sides
Is to the Sine of half the ſum of the Sides,
So is the Tangent of half the difference of the Angles,
To the Co-tangent of half the Angle required.

Case 3.

Two Sides with the Angle between them being given, to find the other Side.

Fig. 3.

For the ſolution of this and all the following Caſes, except the two laſt, I ſhall ſuppoſe the oblique angular Triangle to be reduced into two right angles by a Perpendic. let fall from an Angle to the oppoſite Side or Baſe continued if need require; by which means it will ſometimes fall within the Triangle, and ſometimes without.

☞ *Note*, The Perpendicular muſt always fall from the Extremity of a known ſide, and oppoſite to a known Angle annexed to that Side, ſometimes conterminate with, or incident on a Side required.

Let the Side A B, be 74 *d.* and the Side A C, be 50 *d.* and the Angle at A, 40 *d.* be given, to find the Side B C. In this Caſe the Perpendicular may fall from the Extremity of either Side, but oppoſite to the Angle given:

Example.

The Perpendicular in Fig. 3. falls from the Extremity of the Side A C, and upon the Baſe A B,

A B, and oppoſite to the angle given A, and ſo reduces the oblique Triangle A B C, into two right Angles, (*viz.*) the right angled Triangles, A D C, and B, D, C.

And firſt, for the Baſe A D, in the Triangle A D C, right angled at D, you have the Hypothenuſa A C, 50 *d.* and the Angle at the Baſe A, 40 *d.* then find the Baſe A D, by the firſt Caſe of right angled Spherical Triangles. Thus;

To the Co-ſine of the middle part A,
40 *d.* 00 *m.* ——— 9884254
Add the Radius D, 90 *d.* 00 *m.* ——— 10000000

And from their Sum, which is — 19884254

Subſtr. the Co-tang. A C, 50 *d.* 00 *m.* — 9923814

And there remains the Tangent A D,
42 *d.* 24 *m.* ——— 9960440

Subſtract the Baſe A D, 42 *d.* 24 *m.* from the Baſe A B, 74 *d.* and there will remain the Baſe B D, 31 *d.* 36 *m.*

When the Perpendicular falls within the Triangle, this Baſe A D, muſt be ſubſtracted from the baſe A B, and the Remainer will be the baſe B D; but if the Perpendicular falls without the Triangle, the baſe C D,

muſt be added unto the baſe A C, and their Sum will be the baſe A D. Fig. 4. Then ſay,

As the Coſine of the baſe A D,
42 *d.* 24 *m. C. A.* ——— 0131676
Is to the Co-ſine of the Hypothenuſa A C, 50 *d.* ——— 9808067 } Add.
So is the co-ſine of the baſe B D,
31 *d.* 36 *m.* ——— 9936300

To the Co-ſine of the Hypothenuſa B C, 41 *d.* 16 *m.* ——— 9876013

According to the Fifth Concluſion.

The Second Example.

Caſe 3.

Let the Side A C, be 50 *d.* and the Side B C, be 51 *d.* 18 *m.* and the Angle at C, be 106, complement to 180, the leſſer Angle C, 74 *d.*

Fig. 4.

In this Example the Perpendic. falls without the Triangle, and you have in the Triangle B D C, right angled at D, the Hypothenuſa B C, and the leſſer angle C, which

is

is 74 *d.* to find the Baſe D C; then by the firſt Caſe of right angled Spherical Triangles,

To the Co-ſine of the middle part
C, 74 *d.* ——— 9440338
Add the Radius 90 *d.* ——— 10000000

And from their Sum, which is —— 19440338
Subſtract the Co-tangent of B C,
51 *d.* 18 *m.* ——— 9903714

And there will remain the Tang.
D C, 20 *d.* 7 *m.* ——— 9536624

Now becauſe the Perpendicular falls without, add the baſe D C, unto the Side A C, and their ſum will be the baſe A D, thus; add 20 *d.* 7 *m.* unto A C, 50 *d.* their ſum is A D, 20 *d.* 7 *m.* Then by the Fifth Concluſion, ſay thus;

As the Co-ſine of the baſe D C,
20*d.* 7 *m. Co. Ar.* ——— 00027338
Is to the Co-ſine of the Hypothenuſa,
B C, 51 *d.* 18 *m.* ——— 9796049 } Add.
So is the co-ſine of the baſe A D,
70 *d.* 7 *m.* ——— 9531614

To the co-ſine of the Hypothen.
A B, 76 *d.* 55 *m.* ——— 9355001

Which is the Side required.

Caſe

Case 4.

Two Angles and a Side opposite to one of them, being given, to find the Side comprehended by those Angles, if it be known, whether the Side sought, or the Side opposite to the other given Angle be more or less than a Quadrant.

Fig. 5.

In this Case there are two Varieties; one when the Perpendicular falls within the Triangle, and the other is when the Perpendicular falls without.

☞ *Note*, If the given Angles are acute, the Perpendicular falls within; but if one is acute, and the other obtuse, the Perpendicular falls without: The Perpendicular in this Case must fall from the Extremity of the given Side on the Side enquired, if need be.

Example 1.

Let the Angle at A, be 40 *d.* and the Angle at B, be 56 *d.* and the Side opposite unto the Angle B, (*viz.*) A C, be 43, and let A B, be required.

Here

Here the given Angles being acute, the Perpendicular falls within, and reduces the Triangle A B C, into two right Angles, *viz.* the right angled Triangle A D C, right angled at D, and the right angled Triangle B D C, right angled at D; alſo the Baſes A D, and B D, are to be found in both.

Firſt, For the Baſe A D, by the firſt Caſe of right angled ſpherical Triangles:

Thus;

To the Co-ſine of the middle part A, 40 *d.* ——— 9884254
Add the Radius 90 *d.* ——— 10000000

And from their ſum, which is —— 19884254
Subſtract the Co-tangent of A C, 43 *d.* ——— 10030344

And there will remain the Tangent of A D, which is 35 *d.* 32 *m.* —— 9853910

For the Baſe B D, by the 6.th Concluſ. ſay,
As the Tangent of the Angle B, 56 *d.* *Co. Ar.* ——— 9828987
Is to the Sine of the baſe, A D, 35 *d.* 32 *m.* ——— 9764308
So is the Tangent of the angle A, 40 *d.* ——— 9923814
To the Sine of the Baſe B D, 40 *d.* 59 *m.* ——— 9116809

Add.

Now becauſe the Perpendicular falls within the ſum of the Baſes A D, and D B, is the Side A B required.

The Second Example.

Caſe 4.

Let the Angle *A*, be 40 *d.* and the angle at C, be 106 *d.* and the Side oppoſite unto the angle C, be 64 *d.* and let the Side A C be demanded.

Fig. 6.

Here one angle is acute, and the other obtuſe; therefore the Perpendicular falls without, and by that means there is the Triangle B D C; which is as it were added unto the Triangle A C B, to make it into two right angled Spherical Triangles *viz.* the right angled Triangle, A D B, and the right angled Triangle, C D B; the baſe A D, and C D, are to be found.

And firſt, for the Baſe A D, by the firſt Caſe of Spherical Triangles: Thus;

Unto

Unto the Co-ſine of the middle part
A, 40 *d.* ——— 9884254
Add the Radius 90, ——— 10000000

And from their ſum, which is — 19884254
Subſtract the Co-tangent of A B,
64 *d.* ——— 9688182

And there will remain the Tangent
A D 57 *d.* 31 *m.* ——— 10196072

Then for the Baſe D C, by the Sixth Concluſion, ſay,

As the Tangent of the Angle C, 106, Complement to 180, is 74 *d. co. ar.* 9457496
Is to the Sine of the baſe A D,
57 *d.* 31 *m.* ——— 9926110 } Add.
So is the Tangent of the Angle
A, 40 *d.* 00 *m.* ——— 9923814

To the Sine of the baſe D C,
11 *d.* 42 *m.* ——— 9307420

Now by reaſon the Perpendicular falls without the Triangle, the difference of the two baſes is the ſide A C. Thus;

Subſtract the baſe D C 11 *d.* 42 *m.* from the baſe A D, 57 *d.* 31 *m.* and there will remain the ſide A C, 45 *d.* 30 *m.* Caſe

Case 5.

Fig. 5.

Two Angles and a Side opposite to either of them, given to find the other Angle, if it be known, whether the Angle enquired or the Side opposite to the other Angle given, be more or less than a Quadrant.

☞ *Note*, The Perpendicular must fall from the enquired Angle.

Example 1.

Let the angle at A, be 40 deg. and the angle at B, be 56 deg. and the side AC, 43 deg. and let the angle C be required.

In this Case also, because the given angles are acute, the Perpendicular falls within, and reduces the Triangle A B C, into two right angles, *viz.* the right angled Triangle ADC, right angled at D, and the right angled Triangle B D C, right angled at D. Also the Vertical angles A C D, and B C D, are to find: which you may do by the Sixth Case *Right Angled Spherical Triangles:*

Thus,

The

The co-ſine of the middle part A C,
43 *d.* —— 9864127
Unto which add the Radius 90, — 10000000

And from their ſum, which is —— 19864127
Subſtract the co-tangent of the angle
A, 40 *d.* —— 10076186

And there will remain the co-tangent of ACD, which is 58 *d.* 28 *m.* — 9787941

Then for the Vertical Angle BCD, by the Fourth Concluſion, ſay,

As the co-ſine of the angle at the baſe
A, 40 *d. Co. Ar.* —— 0115746
Is to the ſine of the firſt Vertical
Angle, A CD, 58 *d.* 28 *m.* — 9930611
So is the co-ſine of the angle at
the baſe B, 50 *d.* —— 9808067
To the ſine of the ſecond Verti-
cal angle BCD, 45 *d.* 40 *m.* — 9854424

} Alt.

The Perpendicular falling within, the firſt Vertical angle A C D, 58 *d.* 28 *m.* muſt be added unto the ſecond Vertical ang. B CD, 45 *d.* 40 *m.* and their Sum will be the angle C, 104 *d.* 8 *m.* which was required.

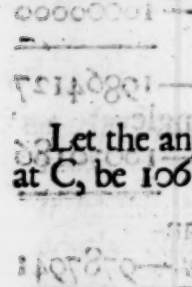
Caſe

The Second Example.

Case 5.

Let the angle at A, be 40 *d.* and the angle at C, be 106 *d.* and the ſide A B, be 64 deg.

Fig. 6.



Here the Perpendicular falls without, and alſo makes two right angles, *viz.* the *Right Angled Triangle* A D B, and the *Vertical Angles* A B D, and C B D, are enquired.

And firſt, for the *Vertical*, A B D, by the Sixth Caſe. ——— Thus:

To the co-ſine of the middle part A B, 64 *d.* ——— 9641842
Add the Radius 90, ——— 10000000
And from their Sum, which is —— 19641842

Subſtract the co-tangent of the angle A 40 *d.* ——— 10076186
And there will remain the co-tang. of A B D, 69 *d.* 48 *m.* ——— 9565656

Then for the *Vertical Angle* C B D by the Fourth Concluſion; ſay,

As

As the co-ſine of the angle at the Baſe A 40 *d* Co. *Ar.* ——— 0115746
Is to the ſine of the firſt vertical angle A B D, 69 *d.* 48 *m.* —— 9972431
So is the co-ſine of the angle at the baſe C, 106 the comp. to 180. 74 *d.* 9440338
} Add.

To the Sine of the ſecond vertical angle C B D, 49 *d.* 44 *m.* —— 9528515

The Perpendicular falling without the Triangle, the difference of the Vertical is the angle B, Thus;

From 69 *d.* 48 *m.* ſubſtract 19 *d.* 44 *m.* and there will remain 50 *d.* 4 *m.* for the angle B.

Caſe 6.

Two Sides and an Angle oppoſite to either of them given, to find the other ſide.

Fig. 7.

If it be known whether the ſide enquired, or the ſide oppoſite to the other given angle be more or leſs than a Quadrant.

This Caſe hath alſo two Varieties; one, when the given angle is acute, then the Perpendicular falls within the Triangle; the other when

when the given angle is obtuſe, and then the perpendicular falls without.

Example 1.

Let the angle at B, be 40 *d.* and the Side A C 50 *d.* and the Side B C, be 43 *d.*

Note, The Perpendicular falls from the concourſe of the Given Sides on the Side enquired, continued if need require:

As for Example.

In *Fig.* 7. the Perpendicular falls from the meeting of the two Given Sides at C, and reduces the Triangle A B C, into two right angles, *viz.* the right angled Triangle ADC, and the right angled Triangle B D C, in both which A D, and B D, is to find.

And firſt for the Baſe B D, by the firſt Caſe of *Right Angled Spherical Triangles*:

Thus;

To the Co-ſine of the middle part B, 40 *d.*	9884254
Add the Radius 90 *d.*	10000000
And from their Sum which is	19884254
Subſtract the co-tangent of BC, 43 *d.*	10030344
And there remains the Tangent of B D 35 *d.* 32 *m.*	9853910

Then

Then for the Baſe A D ; By the Fifth Concluſion.

As the co-ſine of the Hypothenuſa
B C, 43 *d. co. ar.* ——— 0135873
Is to the co-ſine of the baſe B D,
35 *d.* 32 *m.* ——— 9910506 } Add.
So is the co-ſine of the Hypothenuſa A C, 50 *d.* ——— 9808067

To the co-ſine of the baſe A D,
44 *d.* 20 *m.* ——— 9854446

The Perpendicular falling within the Triangle, the ſum of both the baſes is the ſide required : thus;

The baſe B D, 35 *d.* 32 *m.* being added unto the baſe A D, 44 *d.* 20 *m.* their ſum will be the ſide A B, 79 *d.* 52 *m.*

The ſecond Example.

Caſe 6.

Let the Side A B be 64 *d.* and the Side B C, be 43 *d.* and the angle at C, be 106 *d.*

Fig. 8.

Here the Perpendicular muſt fall from the Concourſe of the Sides A B, and B C, from the angle B, upon the Side A C, continued unto D, and oppoſite unto the leſſer angle C, and ſo reduces the oblique Triangle A B C into two right angles, *viz.* the right angled Triangle B D C, right angled at D, and the right angled Triangle A D B, right angled at D; alſo in the right angle C D B, you have the hypothenuſa B C, 43 *d.* and the leſſer angle C, being the complement to the greater, unto a Semicircle, to find the baſe C D; which you may do by the firſt Caſe of *Right Angled Spherical Triangles*:

Thus,

To the Co-ſine of the middle part C, 74 *d.*	9440338
Add the Radius D, 90 *d.*	10000000
And from their ſum, which is	1944033 8
Subſtract the Co-tangent of B C, 43 *d.*	10030344
And there will remain the Tangent D C, which is 14 *d.* 35 *m.* the baſe required.	9409994

Then

Then for the Base A D, by the 5th. Concluſ. ſay,

As the co-ſine of the Hypothenuſa
B C, 43 *d.* 9 *m.* ——— 0135873
Is to the co-ſine of the baſe,
D C, 14 *d.* 25 *m.* ——— 9986104 } Add.
So is the co-ſine of the hypothenuſa A B, 64 *d.* ——— 9641842

To the co-ſine of the baſe A D,
54 *d.* 31 *m.* ——— 9763819

Becauſe the Perpendicular falls without the Triangle, ſubſtract the baſe B C from the baſe A D, and there will remain the ſide A C, required: Thus;

Subſtract D C, 14 *d.* 25 *m.* from the baſe A D, 54 *d.* 31 *m.* and there will remain the Side A C, 40 *d.* 6 *m.*

Caſe 7.

Two Angles and a Side being given, to find either of the other Sides.

Fig. 9.

In this Caſe the Perpendicular muſt fall from the concourſe of the Sides given and ſought, on the Third, continued if need require.

This Caſe alſo hath two Varieties: The firſt is, when the Perpendic. falls within the Triangle, and one of the given angles obtuſe: the other is, when the Perpendicular falls without, and both the given angles are acute.

Example 1.

Let the ſide B C, be 43 deg. and the angle at B, be 46 deg. and the angle at C, be 106 deg. and let the ſide A C, be required.

Here the Perpendicular falls from the angle C, upon the baſe A B, and reduces the oblique angled Triangle into two right, *viz.* the right angled Triangle B D C, right angled at D, and the right angled Triangle A D C, angled at D: Alſo the angle at the Perpendicular D C B, is to be found by the third Caſe of right angled Spherical Triangles.

Thus;

To the Co-ſine of the middle part B C, 43 *d.*	9864127
Add the Radius 90 *d.*	10000000
And from their Sum, which is	19864127
Subſtr. the Co-tang. of the angle B, 46 *d.*	9984837
And there remains the co-tangent of the angle D C B, 52 *d.* 52 *m.* required	9879290

This

This 52 *d.* 52 *m.* because the Perpendicular falls within, must be substracted from the angle C, 106, and there will remain the angle A C D, 53 *d.* 8 *m.* Then for the hypothenusa A C, by the 7th. Conclusion.

Thus;

As the co-sine of the angle A C D, 53 *d.* 8 *m.* *co. ar.* ———— 0221881
Is to the tangent of the Hypothenusa, B C 43 *d.* ———— 9969656
So is the co-sine of the angle A C D, 52 *d.* 52 *m.* ———— 9780801

Add.

To the tangent of the hypoth. A C, the side required, 43, 10.—9972338

The second Example.

Case 7.

Let the base A B, be 64 *d.* and the angle at A, be 40 *d.* and the Angle at B, be 56 *d.* and the Side B C required.

Fig. 10.

Here the Perpendicular falls from the angle B, upon the Side A C; but if the Side A C, had been required, the Perpendicular would

fall from the angle A, upon the Side BD: by which means the oblique angled Triangle ABC, will be reduced into two right angled Triangles, *viz.* the right angled Triangle CDB, right angled at D, and the right angled Triangle ADB, the angle at the Perpendicular C, is to be found; and the hypothenusa CB.

Also for the angle at the Perpendicular C, by the Third Case of *Right Angled Triangles*: Thus;

To the Co-sine of the middle part AB, 64 *d.*	9641842
Add the Radius 90 *d.*	10000000
And from their Sum which is	19641842
Substract the co-tangent of the angle A, 40 *d.*	10076186
And there remains the co tangent of the angle at the Perpendicle B, required, 69 *d.* 48 *m.*	9565656

From the angle found, substract the angle B, 56 deg. because the Perpendicular falls without the Triangle, and there remains the lesser angle at the Perpendicular CDB, 13 *d.* 48 *m.*

Then

Then for the Hypothenuſa CB, by the 7th. Concluſion, ſay,

As the co-ſine of the angle at the Perpend. B, 13 *d.* 48 *m. Co. Ar.* — 0012721
Is to the tangent of the hypothenuſa A B, 64 *d.* 00 *m.*, — 0311828 } Add.
So is the co-ſine of the angle at the Perpend. B, 69 *d.* 48 *m.* — 9538194 }

To the tangent of the hypothen. C B, required, 36 *d.* 06 *m.* — 9862743

Caſe 8.

Two Angles and a Side comprehended by them, given, to find the other angle.

Fig. 9.

In this Caſe the Perpendicular will fall from the extremity of the given Side, and oppoſite to the angle enquired.

This Caſe hath alſo two Varieties; one, when the Perpendicular falls within, and the other when the Perpendicular falls without.

Example 1.

Let the Side B C, be 43 *d.* and the angle at B, be 46 *d.* and the angle at C, be 106 *d.* and let the angle at A be required.

The Perpendicular falls from the angle C, oppoſite to the angle at A, and alſo oppoſite to the given angle B, upon the baſe A B; and ſo reduces the oblique angle A B C, into two right angles, *viz.* the right angled Triangle B D C, right angled at D; and the right angled Triangle A D C, right angled at D.

Alſo, the firſt Operation is to find the angle at the Perpendicular C, in the right angled Triangle B D C; which may be performed by the third Caſe of right angled Spherical Triangles: Thus;

To the co-ſine of the middle part B C, 43 *d.*	9864127
Add the Radius D, 90 *d.*	10000000
And from their Sum, which is	19864127
Subſtract the co-tangent of the given angle B, 46 *d.*	9984837
And there will remain the co-tang. of the ang. at the perp. D, 53 *d.* 52 *m.*	9879290

This

This 52 *d.* 52 *m.* muſt be ſubſtracted from the whole angle C, becauſe the Perpendicular falls within, and there will remain the angle C, in the right angled Triangle A C D.

Then for the angle at the baſe in that ſame Triangle, it being the angle ſought, the Proportion according to the Fourth Concluſion, will be thus;

As the Sine of the firſt angle at the Perp. C, 52 *d.* 52 *m. co. ar.* —0098415
Is to the co-ſine of the angle at the baſe B, 46 *d.* 00 *m.* ——9841771
So is the ſine of the ſecond angle at the perp. C, 53 *d.* 8 *m.* —9903108

To the co-ſine of the angle at the Baſe required, A, 45 *d.* 08.——6843294

Add.

The Second Example.

Caſe 8.

Let the ſide A B be 64 *d.* and the angle at A, be 40 *d.* and the angle at B, 56 *d.* and let the angle at C, be required.

Fig.

Fig. 10.

In this Variety the Perpendicular falls without, from the angle B, oppoſite both to the angle given and required; the ſide A C, being extended to D, to meet the Perpendicular B D.

Then for the greater angle B, by the third Caſe of right angled Spherical Triangles, work thus:

To the Co-ſine of the middle part
A B, 64 *d.* ——————— 9641842
Add the Radius 90 *d.* ——————— 10000000

And from their Sum, which is —— 19641842
Subſtract the Co-tangent of the angle A, 40 *d.* ——————— 10076186

And there will remain the co-tang. of the greater angle B, which was required, 69 *d.* 48 *m.* ——————— 9565656

Now by reaſon the Perpendicular falls without the Triangle, the angle B muſt be ſubſtracted from the greater angle B; and there will remain the leſſer angle at the Perpendicular.

Then for the Hypothenuſa B C, by the Fourth Concluſion, ſay thus:

As

As the ſine of the greater angle at Perp. B, 69 *d.* 48 *m.* *co. ar.* —0027569
Is to the co-ſine of the ang. at the baſe A, 40 *d.* 00 *m.* —9884254
So is the ſine of the leſſer angle at the Perpend. B, 13 *d.* 48 *m.* —9377549

To the co-ſine of the angle at the baſe C, in the leſſer Triangle BCD, 78 *d.* 46 *m.* —9289372

Which, if you ſubſtract from 180 *d.* leaves the angle C, which was required.

Caſe 9.

Two Sides and an Angle oppoſite to either of them, being given, to find the Angle comprehended by them, if it be known whether the Angle enquired, or the Angle oppoſite to the other ſide, be more or leſs than a Quadrant.

Fig. 7.

Let the Side A C, be 50 *d.* and the Side B C, 43 *d.* and the angle at B, 40 *d.*

☞ *Note*, In this Caſe, the Perpendicular muſt fall from the enquired angle, and on the

the unknown Side; and by that means reduceth the oblique angled Triangle A B C, into two right angled Triangles *viz.* the right angled Triangle, A D C, right angled at D, and the right angled Triangle, B D C; right angled at D also. The angle at the Perpendicular is to be found in both: but the angle at the Perpendicular in the Triangle B D C, must be found first by the third Case of right angled Spherical Triangles. Thus;

Example 1.

To the Co-sine of the middle part B C, 43 *d.*	9864127
Add the Radius 90,	10000000
And from their sum, which is	19864127
Substract the Co-tangent of the Extream B, 40 *d.*	10076186
And there will remain the co-tangent of the angle at the Perpendicular C, 58 *d.* 28 *m.*	9787941

As

Then by the Seventh Concluſion, ſay,

As the Tangent of the Side A C, 50 *d.*
Is to the co-ſine of the firſt angle C found, 58 *d.* 28 *m.*
So is the Tangent of the Side B C, 43 *d.* 00 *m.*
To the co-ſine of the ſecond angle C.

Theſe two Angles C being added together, give the great Angle C.

The Second Example.

Let the Side A B, be 64 *d.* and the Side A C 50 *d.* and the Angle B 40 *d.* and let the Angle A be demanded.

The Perp. alſo muſt fall from the enquired angle A, upon the unknown Side B C; and by that means the Perpendicular falls without, and doth reduce the oblique Triangle A B C, into two right ones, *viz.* the right angled Triangle B D A, and the right angled Triangle A D C, right angled at D, and the angle at the Perpendicular A, muſt be found in both; but you muſt find the angle at the Perpendicular A, in the right angled Triangle B D A, by the third Caſe of right angled Triangles:

Thus:

To

To the co-ſine of the middle part
B A, 64 *d.*———— 9, 641842
Add the Radius 90 *d.*———— 10, 000000

From their Sum which is ———— 19641842
Subſtract the co-tangent of the
Extream B, 40 *d.*———— 10076186

There will remain the co-tangent
of the angle at the Perpendicu-
lar A, 69 *d.* 48 *m.*———— 9565656

Then, as before, by the Seventh Concluſion, ſay,

As the Tangent of the Side A C,
50 *d. co. ar.*———— 9923814
Is to the co-ſine of the firſt angle
A, found, 69 *d.* 48 *m.*———— 9538194
So is the Tangent of the Side A B,
64 *d.* 00 *m.*———— 0311818

To the co-ſine of the ſecond angle
A, 53 *d.* 33 *m.*———— 9773826

The ſecond angle A, 53 *d.* 33 *m.* being ſubſtracted from the firſt angle A, 69 *d.* 48 *m.* leaves the angle A, 16 *d.* 15 *m.* deſired, becauſe the Perpendicular falls without.

CASE

Case 10.

Fig. 7.

Two Sides and the Angle comprehended by them, given, to find either of the other Angles.

Example 1.

Let the side A C, be 50 *d.* and the side A B, be 64 *d.* and the angle at A, 40 *d.* to find the angle B.

Note, Here in this first Example, the Perpendicular must fall within from the unknown angle, and opposite to the angle demanded, and by that means reduceth the oblique Triangle A B C, into two right angles, *viz.* the right angled Triangle A D C, right angled at D, and the right angled Triangle B D C, right angled at D: and in the first right angled Triangle A D C, you have the Hypothenusal A C, 50 *d.* and the angle at the base A, 40 *d.* to find the base A D, by the first Case of right angled Triangles; thus:

To

To the co-ſine of the middle part
A, 40 *d.* ——— 9884254
Add the Radius D, 90 *d.* ——— 1000000

From their Sum, which is ——— 19884254
Subſtract the co-tangent of the
Extream A C 50 *d.* ——— 9923813

And there will remain the tangent
of the Extream A D, 42 *d.* 24 *m.* — 9960441

Now becauſe the Perpendicular falls within the Triangle, therefore you muſt ſubſtract the Extream A D 42 *d.* 24 *m.* from the ſide A B, 64. and there will remain the baſe B D, 21 *d.* 36 *m.* Then by the Sixth Concluſion, ſay,

As the ſine of the baſe B D, 21 *d.*
36 *m. co. ar.* ——— 0434006
Is to the tangent of the angle given
A 40 *d.* ——— 9923813
So is the ſine of the baſe A D, 42 *d.*
24 *m.* ——— 9828855

To the tangent of the angle de-
manded B, 56 *d.* 57 *m.* ——— 10186674

The Second Example.

Fig. 10.

Let the angle B, be 40 *d.* and the ſide A B, 64 *d.* and the ſide B C, 40 *d.* and let the angle C, be demanded.

Now the Perpendicular in this Variety falls from the angle A, upon the ſide B C, continued to D, and the angle D C A in the Triangle C D A, right angled at D; it being one of the two right angles that the Perpendicular maketh, the other will be the Triangle B D A, right angled at D, in which there is given the Hypothenuſa B A, 64 *d.* and the angle at the baſe B, 40 *d.* to find the baſe B D, by the firſt Caſe of Right angled Triangles: Thus;

To the co-ſine of the middle part B, 40 *d.*	9884254
Add the Radius D, 90 *d.*	10000000
From their Sum which is	19884254
Subſtract the cotangent of the Extream B A, 64 *d.*	9688182
And there will remain the tangent of the Extr. B D, 57 *d.* 31 *m.*	10196072

Now becauſe the Perpendicular falls without, you muſt ſubſtract the ſide BC, 40 *d.* from the ſide BD, 57 *d.* 31 *m.* and there will remain the baſe CD, 17 *d.* 31 *m.* of the right angled Triangle CDA, right angled at D.

Then by the Sixth Concluſion ſay,

As the ſine of the baſe CD, 17 *d.*
31 *m.* 00 *ar.* 0521458
Is to the tangent of the angle given
B 40 *d.* 9923813
So is the ſine of the baſe BD, 57 *d.*
31 *m.* 9926109
To the tangent of the angle C,
66 *d.* 58 *m.* 10371380

in the Triangle CDA, which being ſubſtracted from 180 *d.* leaves the angle C, in the Triangle ABC, which was required;

Thus:

180 00
Subſtr. the ang. C. 66 58 in Triang. CDA.
the ang. C. 113 2 in Triang. ABC.

Caſe

Case 11.

The Three Sides given, to find an Angle.

Fig. 11.

In this Case and the next, there is no need of letting a Perpendicular fall from any angle: But the Rule to find an angle, is thus;

As the Rectangular Figure of the sines of the side comprehending the enquired angle,
Is to the Square of the Radius,
So is the Rectangle of the sines of half the Sum, and half the difference of the base, and difference of the sides,
To the Square of the sine of half the angle required.

Example 1.

Let the side A B, be 74 *d.* and the side A C, 50 *d.* and the side B C, 43 *d.* and let the angle C, be required.

☞ *Note*, You must let the side that is opposite to the angle demanded, be always first in the operation, and then the Rule is,

Add the three ſides together; and out of half their Sum ſubſtract every ſide, ſo have you the differences, the Arithmetical Complements of the Logarithms of half the Sum, and difference of the baſe added unto the Logarithms of the difference of the other two ſides give a number, the half whereof is the Tangent of half the Angle deſired:

Thus;

The Side	A B	74 00	
The Sides	A C	50 00.	
	B C	43 00.	
The Sum ——		167 00.	
		Sines.	
The half Sum ——		83 30.0002801	*co. ar.*
The diff. of the baſe.		9 30.0782391	*co. ar*
The diff. of the ſide AC		33 30.9741889	Add.
The diff. of the ſide BC		40 30.9812544	
		20339625	
		10169813	is th

Tangent of half the Angle 55 *d.* 56 *m.*
The double is 111 52 the angle at C.

The Second Example.

Let the Angle at A be required, then the ſide B C muſt be firſt in the Operation.

Thus

Thus,

The ſide B C———	43 00.		
The ſide A B———	74 00.		
The ſide A C———	50 00.		
		Sines.	
Their ſum is———	167 00.		
Their half ſum is—	83 30.	0002801	*co. ar.*
The diff. of the baſe	40 30.	0187456	*co. ar.*
Diff. of the ſide A B	9 30.	9217609	Add.
Diff. of the ſide A C	33 30.	9741889	
The ſum of the ſines		1949755	
The half ſum is———		9574277	
the Tangent of —	20 35		

Which being doubled, is the Angle at A, required——— 41 10.

The Third Example.

Let the Angle at B, be required, then the ſide A C, muſt be firſt in the operation:

Thus;

The ſide A C	50.		
The ſide A B	74.	Add,	
The ſide B C	43		
Their Sum is	167 00	Sines.	
Their half Sum	83 30	—— 0002801	*co. ar.*
The diff. of the baſe and half Sum	33 30	—— 0258111	*co. ar.*
The diff. of the ſide A B, and half Sum	9 30	—— 9217609	Add.
The diff. of the ſide B C, and half Sum,	40 30	—— 9812544	
The ſum of all theſe Sines is		19291065	
The half whereof is		—— 9645534	

The Tangent of 23 51

Which doubled, is 47 42. the Angle required.

Caſe

Caſe 12.

Three Angles of an oblique Spherical Triangle being given, to find any of the Sides.

Fig. 12.

Let the angle at A, be 41 *d.* 19. and the angle at B, be 47 *d.* 42 *m.* and the angle at C, 111 *d.* 52 *m.* and let the ſide A B be firſt required. This Caſe is but the Converſe of the former, the ſides being turned into angles, and the angles into ſides; wherefore let the angle oppoſite to the ſide required, be always firſt in the Operation: then the Rule will be the ſame, *viz.* add the three angles together, and note their Sum, and take half thereof; from half which ſum ſubſtract every angle: ſo have you the difference of every angle, with the half ſum of the angles.

Then if you add Arithmetical Complements of the ſines of the half Sum, and the difference of the angle oppoſite to the ſide required, unto the ſines of the difference of the other two angles with the half Sum: The half Sum of all theſe ſines being added together, will be the tangent of an Arch, which being doubled, will be the ſide required.

Example 1. *for* A B.

The Angle C 111 52,
Compl. to 180 is 68 8.
The angle at B, is 47 42.
The angle at A, is 41 10.
Their Sum is —— 157 00. Sines.
The half sum is 78 30. — 0008807 *co. ar.*

The Diff. of C and
the half sum is, 10 22. — 0744856 *co. ar.*
Diff. of A, & ½ sum, 37 20. — 9709306 Add.
Diff. of B, & ½ sum, 30 48. — 9782796

The sum of these Sines, —— 20245765
The half sum is the Co-tangent
of an Arch, — 37 00. — 10122882

Which doubled, is 74, the side A B required.

The Second Example. For A C.

The Angle B, 47 42
The Angle at C, 68 8, *viz.* the complem.
to 180 *d.*
The Angle at A, 41 10.

Their Sum is 157 00. Sines.
The half Sum is 78 30. — 0008807 *co. ar.*
Diff. of B & half sum 30 48 — 0290694 *co. ar.*
Diff. of C & half sum 10 22 — 9255144 Add.

Diff.

Diff. of A & half sum 37 20 — 9782796

The sum of these sines ——— 19337441
The half sum is the Tangent of half the side required, ——— 25 00. — 19668720
Which being doubled, is the side A C required. ——— 50 00.

The Third Example.

Let the Side B C be required, then the Angle A must be the first in the Operation: Thus;

The Angle at A,	41 10.	
The Angle at C,	68 8.	*viz.* the compl. to
The Angle at B,	47 42.	(180 deg.
Their Sum is,	157 00.	
Their half sum is	78 30.	0008807 *co. ar.*
Diff. of the Angle A, and half sum,	37 20.	0217204 *co. ar.*
Diff. of the Angle C, and half sum,	10 22.	9255144 Add.
Diff. of the Angle B, and half sum,	30 48.	9709306
The sum of these Sines is,		19190461
The half sum is the Tangent of half the side required. ———	21 30.	9595230

Which doubled is 43 00, the side B C, required.

Note,

Note, In ſome of the Caſes before-going, the Affection of the Angle ſought cannot be determined from what is given: Such Caſes are, when the given Angle is acute, and the oppoſite ſide leſs than a Quadrant, and the adjacent, or other ſide greater than the oppoſite ſide; and its Complement to 180 deg. alſo greater than the oppoſite ſide.

So likewiſe, when the given Angle is acute, and the oppoſite ſide is greater than a Quadrant, and alſo greater than the other ſide, and its Complement to 180 degrees.

Note, When Caſes are thus doubtful, there can be but a double Anſwer, and both true; wherefore find the acute Angle, and its Complement to 180 deg. Such Caſes are the Second, Fifth and Ninth.

The other three doubtful Caſes are when the Side ſought cannot be determined from what is given:

Such Caſes are, when the given Angle is acute, and the oppoſite ſide is leſs than a Quadrant, and the other Angle greater than the former given Angle, and its Complement to 180 degrees, being alſo greater, than the ſaid Angle: As alſo, when the given Angle is obtuſe, and the oppoſite ſide greater than a Quadrant, the other Angle being leſs than this Angle, and its Complement to 180 deg. alſo leſs than this Angle; ſuch Caſes are the

First,

First, Fourth and Sixth. The Determination of all other Cases are certain: But seeing these six Cases have passed without Caution in most of our English Books, take the Demonstration as it is Translated from *Clavius Theodosium*, by the Learned Mr. *Collins*, and Published by him in his *Sector* on a Quadrant.

Let A D, and A C be two equal Sides, including the Angle D A C, and both of them less or greater than a Quadrant. *Fig.* 13.

Draw through the Point C and D, the Arch of a great Circle C D, continue it, and draw thereunto another Arch or Side from A, namely, A B; neither through the Poles of the Arch C D, nor through the Poles of the Arch A D, so that the angle B, and B A D may not be right angles, nor the angle A D B: if then each of these sides A D, A C be less than a Quadrant, the two angles C, and A D C, will be acute; and if these arches be greater respectively than a Quadrant, the two angles C, and A D C, will be obtuse, whence it comes to pass that the angle A D B is obtuse, when the angle A D C is acute, and the contrary.

Now forasmuch as the side A C and A D, are equal to each other, the other *Data* (*viz.*) the side A B, and the angle at B, are common to both; for in each Triangle A B D, and A B C, there is given two sides with the angle

at B, oppoſite to one of them. Now this is not a ſufficient *Data* to find the angle oppoſite to the other ſide, which may be either the angle acute at C, or the obtuſe angle A D B, , the complement thereof to a Semicircle: Nor to find the third ſide, which may either be B D, or the whole ſide B C, nor the angle included, which may be either B A D, or B A C, therefore in theſe three Caſes we have required the quality of the angle oppoſite to the other given Side A B, and though it be not ſo much obſerved in the other Trigonometry by Perpendiculars let fall without the knowledge of the ſaid angle, it could not be determined whether the perpendicular will fall within or without the Triangle, nor whether the angle found in the firſt Caſe, be the thing ſought, or its Complement to 180, nor whether the angle or ſegment found by the firſt and ſecond Operation in the other Caſes, are to be added together, or ſubſtracted from each other, to obtain the ſide or angle ſought.

So alſo two angles with a ſide oppoſite to one of them, are not ſufficient *Data* to obtain a fourth thing in the ſaid Triangle without the Affection of the ſide oppoſite to the other given angles. *Fig.* 14.

Let A B, and A C, be two unequal ſides, containing the angle B A C, both together equal

Fig. 1.

Fig. 2.

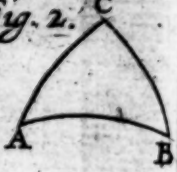


Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

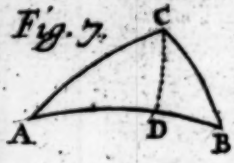


Fig. 8.

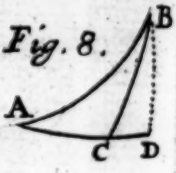


Fig. 9.

Fig. 10.

Fig. 13.

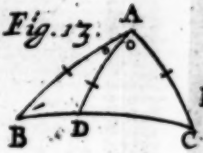
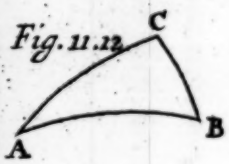


Fig. 14.

Fig. 11. 12.



At the end of the Book

equal to a Semi-circle; one being greater, the other leſſer than a Quadrant: draw through the point B and C, the arch of a great Circle B C, continue it, and draw thereto another ſide A D, but not through the Poles of A C, nor through the Poles of B C, ſo that the angles D, and C A D, may not be right angles, nor the angle A C D a right angle; for if it were a right angle, the angle A B C, whereto it is equal, ſhould alſo be a right angle, and ſo the two ſides A B and A C, by reaſon of their right angles at B and C, ſhould be equal, and be Quadrants, contrary to the ſuppoſition: Now the angles of A C D, and A B C, being equal, which is thus proved; Suppoſe the two ſides A B, and B D to be continued to a Semicircle at E, then will the ſaid angle be equal to its oppoſite angle at B, the ſide A C, by ſuppoſition is equal to the ſide A E, the complement of the ſide A B to a Semicircle, but equal ſides ſubtend equal angles, therefore the angle at C, is equal to the angle at B, or at E; which being admitted, retaining the ſide A D, and angle at D, we have another angle oppoſite thereto, either C or B, which are equal, and common to both Triangles: and ſo if the ſides oppoſite to the given angle at D, were ſought, a double anſwer ſhould be given, either the ſide A C, or the other ſide A B is complement

to

to 180, and the interjacent ſide might be be CD, or BD, and the third angle the leſſer, CAB, or the greater, BAD, which is not commonly animadverted.

FINIS.

Advertiſement.

THere is now in the Preſs a ſmall Piece of Dyalling in *Quarto*, written by *John Holwel* Teacher of the *Mathematicks*, which he intends to be *A Key to the whole Art of Arithmetical Dyalling*, in Two Parts: The firſt ſhewing how to make all manner of Regular, and how to reduce all Irregular Dyals to Regular Forms, by means of a new Latitude and new Declination, with the ſeveral Projections, ſhewing where all the Requiſites lie in the Sphear. The Second Part ſhewing the placing of all the Furniture on all manner of Dyals however ſituated, and that Arithmetically, giving their Reaſons Aſtronomically.

VVhere-

Whereunto will be Annexed the Tables of the Altitude of the Sun, for the beginning of his Entrance in each Sign, and for each hour of the Day, for all Latitudes, from one Deg. to Ninety, with Tables of *Azimuth* to each Latitude, and many other Tables, very uſeful in the Art of Shadows, with their Uſe in making all Conceits in Dyalling, too large to be repeated: As alſo the Deſcription and making of the Conceited Dyals that ſtood on the Pyramid in His Majeſties Privy Garden that was broke down of late Years; the whole containing forty Sheets, beſides Copper Cuts, which will be forty, if not more, all the Projections and Dyals being ſuch.

This Book having been much enquired after, and the Tables belonging thereto, having coſt the Author many Years Study and Labour, the framing of them being of that Nature which never were made publick before, he hath for the the better enabling him to Print it, put them forth by Subſciption, as followeth; *viz.*

1. All that Subscribe for one single Book, shall have it bound up in Calves-Leather for 6 s. one half to be paid at the time of Subscription, and the other half at the Delivery of the Book.

2. All that procure Subscriptions for six Books, shall have a seventh *gratis*.

3. All that are Encouragers of the VVork shall have their Names inserted in the said Book,

4. And for further satisfaction to the Subscribers, the Author will not receive any part of the Subscription-Money till the Book be delivered; but shall lie in the hands of those to whom they do Subscribe, for their better security.

The *Proposals* together with the Names of those both in City and Countrey that are appointed to receive *Subscriptions*, may be had at the Authors House on the East-side of *Spittle-Fields*, over against *Dorset-Street*, next Door to a *Glaziers*.

Where also is taught the *Mathematicks* all its Parts, By the said Author.